Detection of corrugation and wheelflats of railway wheels using energy and cepstrum analysis of rail acceleration

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Abstract: Rolling stock and track damage due to localized (wheelflats) and global (corrugation) railway wheel tread defects is extremely serious, and several devices have been developed to detect these defects. An original experimental and numerical procedure for detection of wheel corrugation and wheelflats has been developed and validated; it processes rail acceleration signals collected by using combined energy and cepstrum analysis criteria. The use of cepstrum analysis proved to be particularly useful as it allows the discrimination of wheelflats independently from the presence of other defects, even when their effects are hidden in globally high acceleration levels due to heavy corrugation. A short survey of damage induced by wheelflats is presented; optimal measurement conditions and extensive examples are then detailed. The results are discussed with particular reference to the applicability of the technique developed to automated detection devices.

Keywords: railway wheel tread defects, corrugation, wheelflat, automatic detection, cepstrum analysis

NOTATION

$C_x(\tau)$	power cepstrum function
f	frequency
$\mathscr{F}^{-1}$	inverse Fourier transform
L_{Aeq}	A-weighted equivalent sound pressure level
N	number of samples used in covariance and cepstrum calculations
$R_{xx}(t)$	autocorrelation function
s_i	ith sample of the acceleration time history
$S_{xx}(f)$	autopower spectrum function
Δt	sampling interval
σ^2	covariance function
τ	quefrency

1 INTRODUCTION

Railway wheel tread is subjected to several damage mechanisms that modify its nominal geometry. These shape errors induce high stress both in the vehicles and in the track and increase the emitted noise. Localized surface defects, known as *wheelflats*, are a typical consequence of wheel blocking during braking due to insufficient adhesion between wheels and rails. For disc-braked vehicles, i.e. mainly for passenger coaches, the tread is particularly smooth and then blocking and subsequent slipping are easier; the adoption of anti-blocking systems limits the problem, especially at the higher velocities where adhesion is particularly low. For block-braked vehicles, i.e. locomotives, freight cars and some types of EMUs, roughness is higher but, for freight cars, load variations and heterogeneity of the train composition lead to the practical impossibility of braking correctly at all loads and velocities. Noise emitted by wheelflats is particularly annoying for passengers.

Global surface defects, often indicated as corrugation, have a less standard genesis. Certainly vehicle dynamics and tread surface hardness variations are important parameters at the beginning of the phenomenon, but its growth has still to be completely understood. Wheels pass over many different routes in their life, making it harder to follow the evolution of the damage and to verify relationships between mileage and wear. Corrugation is easily identifiable as it gives very high and continuous vibration and noise levels.

In this work a methodology based on experimental data energy and cepstrum analysis for the automatic detection of defects that pass a predetermined threshold, to be defined on a statistical basis, is presented and validated. Energy analysis is used to estimate the global stress which the rail

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undergoes, while cepstrum analysis, a signal processing technique capable of detecting echoes even in strongly noisy signals, allows the detection of the wheelflats almost independently from the extent of the damage. The results are discussed with particular reference to the optimal design and positioning of automatic detection devices.

2 DAMAGE DUE TO THE WHEELFLATS

Wheelflats and rail joints produce the same effects, are responsible for extremely serious failures in wheelsets and rails, and are often studied and analysed together. Their consequences are always strongly negative for the regularity and the safety of the transport and for the comfort of passengers and of people that live beside railway lines. The importance of wheelflats is amplified because their identification with transducers positioned in a fixed measuring site is very difficult, as the hits happen once per revolution of the wheel ($\sim$ 3 m) with high peak levels but with very little energy content. If the hit does not occur just over the instrumented section, the transducer can miss the impact and the detection device will fail. On the other hand, the instrumentation of axleboxes of all the vehicles is clearly impossible, especially for freight cars. A short survey of the main consequences of wheelflats, under the hypothesis that the line is geometrically perfect, is presented here.

The analysis of the birth and the growth of the wheelflats is described in the references (**1**–**10**); these deeply analyse the phenomenon within European and American railway administrations, hence with very different axle loads, rail sections and track supports. Heavier freight traffic and minor maintenance of the track lead to numerous wheelset and rail failures in the United States of America, allowing statistical analysis on the propagation and effect of different types of defects in different operating conditions.

Wheelflats start with a definite geometrical shape and limited length (2.5–7.5 cm), and their evolution is such that they do not disappear with time. Instead, they generate the so-called *long wavelength wheelflats* (25–75 cm) which have a still more dangerous effect as they act at lower frequencies that are less compensated by inertia forces than sharp peaks due to a freshly generated wheelflat. Therefore, early detection of wheelflats is very important.

It is well recognized that the intensity of the hits due to wheelflats is at a maximum at a speed in the order of 20–40 km/h and that it remains stable beyond 100 km/h (**1**, **2**). This behaviour is related to the relative mobilities of the wheelset and the rail: as a wheelflat arrives on the rail, the 'absence' of material is such that both the wheel and the rail tend to move towards each other. In this case the rail has a lower inertia and a greater stored elastic energy, and it moves upwards while the wheel tends to 'fly' above the irregularity. Beyond the aforementioned velocity, the rail still tends to restore its original position but acquires only little velocity before the wheelflat ends. It is clear that long wavelength wheelflats are more dangerous, as for them a limit velocity beyond which the wheel 'flies' does not exist; moreover, gauges normally used to check for wheelflats cannot identify these defects.

The power spectrum of vibrations induced by wheelflats is concentrated below 1 kHz, where wheel impedance is much greater than that of the rail; it is then sufficient to consider only the elasticity of the rail, which is considered as an equivalent mass of 0.4 times the unitary mass of the rail (**2**). Absolute accelerations up to 900 g (**3**) have been measured on the rail, and their frequency content is such that they easily reach the sleepers and the ballast underneath, accelerating their wear (**1**, **3**, **6**). Ballast accelerations reach 45 g, and peak forces reach more than 800 kN for concrete sleepers (**3**). Rails are subjected to stresses up to 70 MPa above static and thermal loads with the obvious consequence of rail failures, especially at low temperatures (**1**). Bearings mounted in the axleboxes are subjected to loads above 400 kN, occurring in 0.5 per cent of the freight cars (**8**). Loads on bearings mounted on vehicles that run on lines with wooden sleepers are about one-half of those that run on lines with concrete sleepers. Loads always have the same orientation with respect to the bearing races, generating 'brinelling' and causing seals to break. Empty freight cars exhibit higher axlebox acceleration than loaded wagons (**8**).

Subjective analysis shows that people feel that impulsive noise is more annoying by 3 dB(A) with the same L_{Aeq} for crest factor reasons (**9**). Wheelflats noise, particularly impulsive, is very disturbing especially for sleeping and restaurant coaches, and is particularly amplified in tunnels (**1**); it increases linearly with velocity up to the aforementioned critical velocity, beyond which it remains constant. Normally, the presence of wheelflats increases the L_{Aeq} level by 2–4 dB(A), and hence the global annoyance is equivalent to an increase of about 6 dB(A) of L_{Aeq} (**9**). Fukuda *et al.* (**7**) report that the noise emitted by the Shinkansen train was reduced by 10 dB(A) by eliminating wheelflats.

3 EXPERIMENTAL ACTIVITIES

Noise and acceleration measurements were performed during 1995 and 1996 on the passage of trains at a test site on the railway line just outside the Firenze Santa Maria Novella station towards Rome, where all the trains ran at almost equal speed and where there were no macroscopical defects on the rails. They were all passenger trains, and were mainly composed of disc-braked vehicles. Even though measurements had been specifically planned and performed for other applications (**11**, **12**), data collected proved to be valid for a complete analysis of wheel defects.

A rail was equipped with six monoaxial piezoelectric accelerometers (Fig. 1); simultaneous measurements of

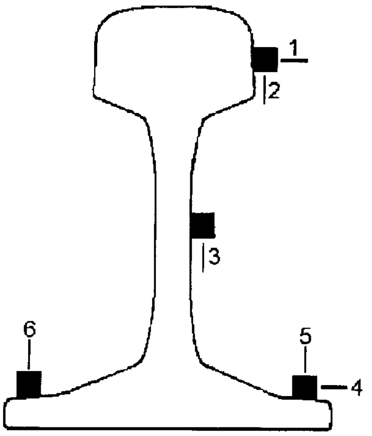

Fig. 1 UIC 60 rail cross-section with accelerometer location and direction of measurements

noise were made with two measurement microphones, but their signals were not used in this work for reasons that will be clarified later. Signals were stored on the hard disk of a personal computer (PC) equipped with a National Instruments general input/output (I/O) acquisition board controlled with LabVIEW software and then off-line processed using MATLAB software. Sampling frequency was 20 480 Hz, but signals were low-pass filtered at 4 kHz before processing, as energy content is higher in this lower frequency range. The different accelerometer signals highlighted a strong deformation of the rail section at around 1 kHz.

Wheelflats detection looks very easy at first glance, as no particular training is required in order to identify their presence at the passage of a train. Unfortunately, this detection is performed by an 'acquisition hardware' (the human ear) that is very sophisticated, since people automatically make a lot of decisions in order to reject or accept a certain signal. Noise due to impacts is easily recognizable at a certain distance from the track axis, but going closer to the rail the local sound emission prevails and the impact lasts only for a short time. For the same reason an accelerometer mounted on the rail is mainly affected by wheel–rail contact forces close to it.

A preliminary survey of recorded signals, which was particularly useful to verify the detection techniques proposed in this work, was then to listen directly to acceleration signals that had been converted to Microsoft Windows WAV files (8 bits, mono) and reproduced with an audio board with a reduced playing frequency (10 240 Hz), or half the recording frequency. This processing slowed train signal reproduction, making it easier to identify peaks due to wheelflats, and lowering the frequencies into the range of greater human ear sensitivity.

As previously mentioned, with a single localized transducer it is impossible to assign a detected wheelflat to a well-defined wheel, as the peaks occur once per revolution (2.5–3 m for a wheel diameter of 800–900 mm), i.e. at a distance that is similar to the rigid pitch of a bogie. With the procedure described here it is possible to detect only the bogie to which a defective wheel belongs, but this does not appear to be a great drawback as the vehicle must be stopped in case of problems and the whole bogie (with its wheelsets) can be easily inspected.

Even if accelerometric measurements are harder to process, standard industrial accelerometers show superior behaviour in comparison to microphones in the very demanding railway environment. Other transducers with better detection capabilities could be designed and used only if their ruggedness proved to be appropriate for applications in this field.

4 CORRUGATION AND WHEELFLATS DETECTION

Signals from all accelerometers have been analysed and compared; accelerometer 2 gave the best results, even if sufficiently good data were obtained from the accelerometers that measured vertical vibration and that were more protected from external agents.

Train speed must be known with a sufficient degree of accuracy, even if, during the tests, no specific device has been used for speed measurement. The analysis of time signals and the knowledge of rolling stock geometrical characteristics (bogie pitch, overall vehicle length) provide a sufficiently accurate speed estimation. Speed determination can be automated with a speed detector fitted in any future industrial version of a device for defect detection.

4.1 Corrugation detection with energy analysis of acceleration signals

This technique compares the energy level of a segment of the signal with one or more predetermined thresholds; these must be carefully defined as energy strongly depends on train speed and on train composition. It is therefore plausible that a set of thresholds has to be defined on a statistical basis.

The energy of a discrete time portion of a signal can be estimated by summing up the squares of the samples or using a similar function available in any statistics software, i.e. the covariance that, for a discrete time signal with N samples, represents the average energy of the N samples of the measured accelerometric signal if it has zero average (this condition is always satisfied as piezoelectric accelerometers cannot measure constant accelerations)

$$\sigma^2 = \frac{\sum_{i=1}^{N} s_i^2}{N-1} \quad \text{where } s_i = s(i\,\Delta t) \quad \text{with } i = 1, 2, \ldots, N \qquad (1)$$

Corrugation detection is carried out by examining the value of the covariance for any segment considered. A fundamental parameter is the choice of N, as too low a value overestimates single peaks and too high a value leads to an excessive reduction of the definition of the signals. To correctly compare the energy level of different trains, the number N of considered samples must be relative to a distance of 1 m run by each train and is thus dependent on train speed, which must be known. Starting the analysis from different times can lead to ambiguous results; to overcome this error, segments have been analysed not consecutively but by using a classical overlap of 87.5 per cent. In this way, the covariance function does not depend on the arbitrary selection of the initial instant of analysis.

The results of the application of this technique to the accelerations measured for four different trains are shown in Fig. 2. Visual inspection of signals is more than sufficient to identify defective bogies correctly, the energy content of which is much higher than the mean value. For example, the ETR450 train has no particularly damaged bogies, while the Intercity train has the first bogie of the restaurant coach very damaged and another bogie (the first of the third first-class coach) that exceeds the average level; for the third and the fourth train similar observations can be made for coaches 4, 5 and 9, and 7 and 8 respectively.

4.2 Wheelflats detection with cepstrum analysis of acceleration signals

The definition of numerical criteria for the identification and quantification of hits due to wheelflats is not a trivial process. The simple comparison of acceleration signals with fixed thresholds can lead to completely mistaken detections, as accelerations are strongly dependent on velocity and axle load (**13**–**16**) and rail–wheel combined surface roughness is responsible for rail vibrations (**17**, **18**): under these conditions it is highly probable that spurious peaks will be measured that are not globally significant and thus to detect non-existent wheelflats. In the same way, detection of wheelflats is impossible using the usual energy threshold comparison, as their peaks affect only a few samples hidden in a random-like signal and the shape of power spectra is not practically influenced by these peaks. An early detection of wheelflats, which must be particularly attractive in the design stage of a new device, must

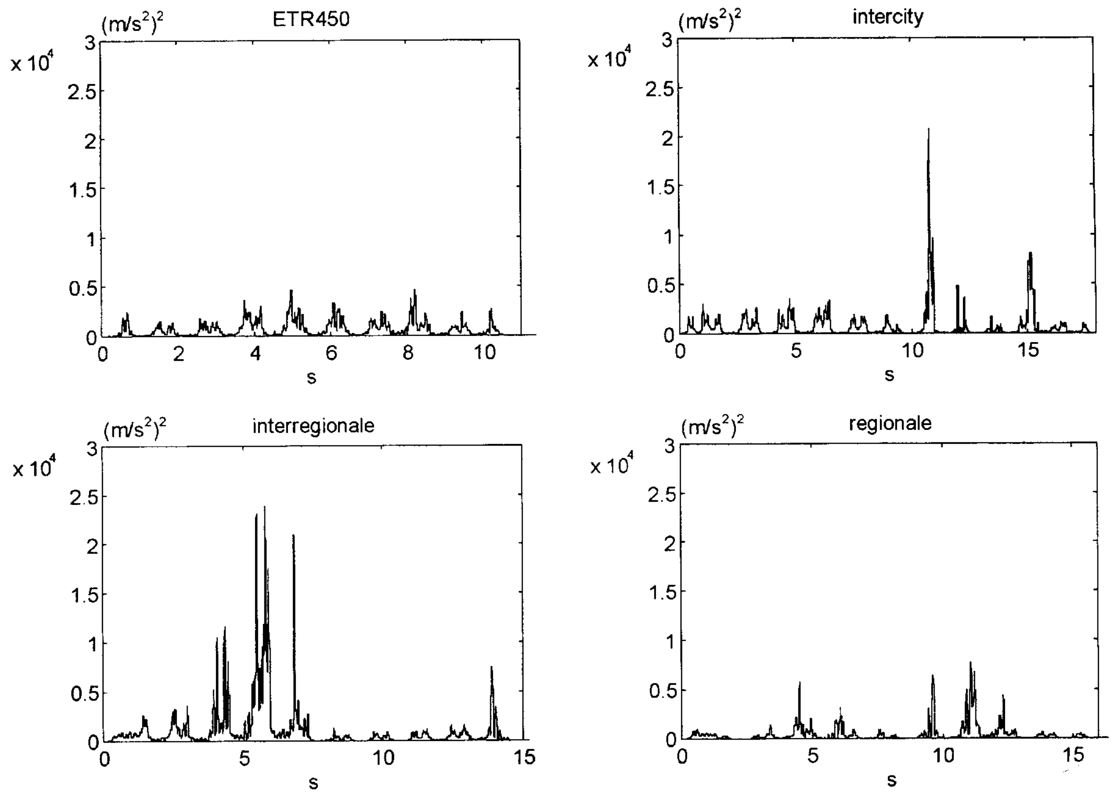

Fig. 2 Energy (covariance) of the segments of signals measured at the passage of trains by accelerometer 2. Trains analysed were the ETR450 (top left, nine elements), the Intercity (top right, locomotive E402 plus six second-class coaches, one restaurant coach, four first-class coaches), Interregionale (bottom left, locomotive E646 plus one mail coach, three first-class coaches, four second-class coaches, one driver coach) and Regionale (bottom right, locomotive E646 plus one mail coach, three first-class coaches, four second-class coaches and one driver coach)

identify even small flats, the effect of which (acceleration) can even be lower than that due to normal roughness of the tread. In this case, no method of energy or threshold analysis can find the defect.

As common signal processing techniques are unable to detect these wheelflats, a rarely used but powerful analysis tool, i.e. cepstrum analysis, has been used in this work. The *power cepstrum* is a discrete time signals analysis function introduced by Bogert *et al.* (**19**) to detect seismic origin echoes; its name arises from the fact that this function is similar to the inverse of the power spectrum (*spec*trum becomes *ceps*trum). It has been used in the analysis of the dynamics of mechanical systems that have a periodic behaviour, such as turbine blades, gears, cutting forces for milling tools and so on. Its fundamental property is the capability to identify and measure those phenomena that have spectral periodicity and side bands. In these cases, it is the best instrument to highlight the presence of acceleration peaks that occur at regular time intervals, as those originated by wheelflats.

The power cepstrum $C_x(\tau)$ is defined as 'the power spectrum of the logarithm of the autopower spectrum of the signal'. The algorithm used for Fourier transform computation is the fast Fourier transform (FFT); the inverse FFT has been used instead of direct FFT in equation (2) as, apart from a scaling factor, it is numerically equivalent to evaluating the direct or the inverse FFT (**20**), while use of the latter makes the cepstrum directly comparable to the autocorrelation function $R_{xx}(t)$

$$C_x(\tau) = |\mathcal{F}^{-1}\{\log[S_{xx}(f)]\}|^2 \tag{2}$$

$$R_{xx}(t) = \mathcal{F}^{-1}[S_{xx}(f)] \tag{3}$$

The analogy between equations (2) and (3) is clear; the independent variable τ, called quefrency, is dimensionally a time and it is related to the independent variable of the autocorrelation function. The advantage of cepstrum is the ability to detect peaks due to echoes, a feature that for autocorrelation strongly depends on the shape of the signal. As excitation due to wheelflats is superimposed on the excitation due to profile roughness, autocorrelation does not sufficiently distinguish the presence of peaks. The reader interested in a general introduction and other definitions of cepstral functions is referred to reference (**21**).

Time histories for cepstrum analysis have been analysed in segments longer than those used for energy computations. To detect the presence of wheelflats it is in fact necessary to process the signal of at least three complete wheel rotations, i.e. a length of more than 9 m, to measure at least three peaks that are then equivalent to 'echoes'; it is not necessary to modify the segment length according to train speed, provided that it remains relatively constant, as under this hypothesis the number of peaks remains the same. The length of a segment must be kept as short as possible, respecting the constraint just mentioned, otherwise the possibility to relate detected damage to a bogie can be lost. The length N should be a power of 2, for direct and inverse FFT computation time reasons.

For each segment the cepstrum is calculated; if its maximum is greater than a defined threshold, the cepstrum itself is analysed to find quefrencies where it passes the threshold. As train speed is known, it is easy to find the rolling distance associated with these 'times', i.e. the number and the relative position of the defects on the wheel tread profile.

The application of cepstrum analysis to recorded signals has given very good results. Figures 3 and 4 show results for all the cases that have been found in two trains

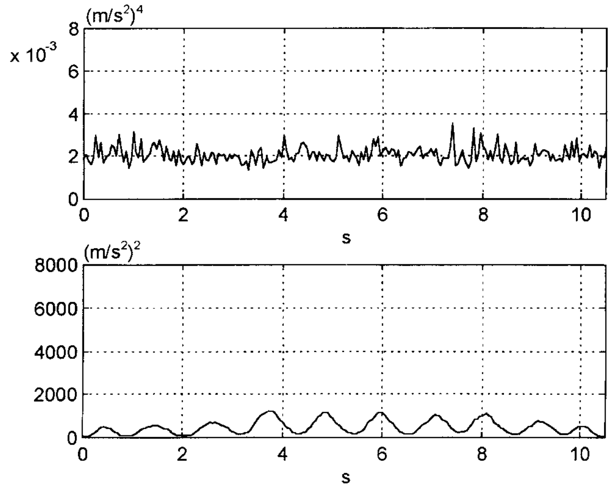

Fig. 3 Cepstrum analysis of the ETR450 train. Maximum of the cepstrum function (top) and energy associated (bottom) for longer signal segments

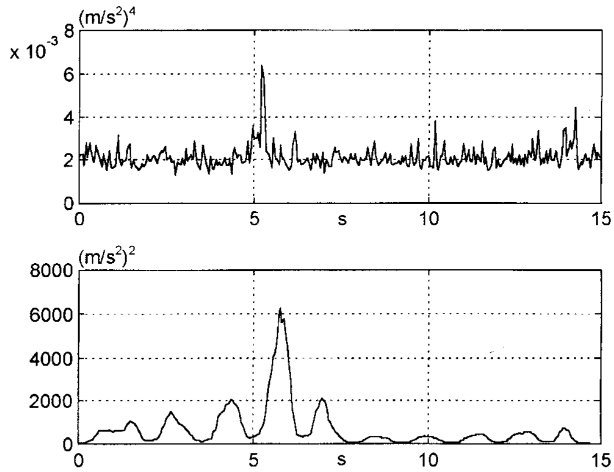

Fig. 4 Cepstrum analysis for the Interregionale train. Maximum of the cepstrum function (top) and energy associated (bottom) for longer signal segments

previously analysed using the energy criterion, i.e. good and worn treads with or without one or more wheelflats.

Cepstrum analysis for the ETR450 train is shown in Fig. 3. The energy has been recalculated for these longer signal segments; the effect of lengthening the segments is similar to a low-pass filtering. As neither energy nor cepstrum maxima are particularly high for the whole train, this train has good treads without wheelflats. Listening to the acoustic reproduction of all accelerometric and noise signals confirms the absence of defects.

The Interregionale train has both good and worn treads with one or more defects (Fig. 4). While the recalculated energy diagram of the fourth coach confirms its high levels, cepstrum analysis shows a very high local maximum slightly before the corresponding energy peak. Observing the acceleration signal (Fig. 5), it is clear how the cepstrum maximum is found in a zone where the signal is not particularly high, and it would be impossible to detect any defect only by direct comparison with thresholds. Where global defects are prevalent it is in fact impossible to detect visually the vibrations due to the wheelflats, that instead can be found *before* the passage of the globally worn zone thanks to cepstrum echo detection capabilities. A zoom in this zone, with the energy level quite low but with a high cepstrum maximum, is shown in Fig. 6. Even if acceleration levels are relatively low (300 m/s^2 against more than 1000 m/s^2 measured a few milliseconds later for the same bogie), it is possible to identify visually in the signal some *pairs* of small peaks. The cepstrum function for this segment shows two clear peaks with quefrencies that are relative to distances of 0.72 m and 2.7 m. This can be due either to two wheelflats on one wheel (in this case the values are the circumferential distance of the defects) or to a wheelflat on each of the two wheelsets of the measured bogie. In any case, a pronounced peak in the cepstrum will be present at a distance equal to the circumference length, and the automatic detection of the presence of one or more wheelflats could be made by comparing the cepstrum value in a range of distances close to 2.7 m to take into account the different wheel diameters and wear. It is worth underlining that this detection would have been impossible with a simple visual inspection or an energy analysis of the signal. The human ear can still hear the peaks due to the wheelflats even if the 'noise' of the signal (i.e. the acceleration due to global defects) is very high, as the

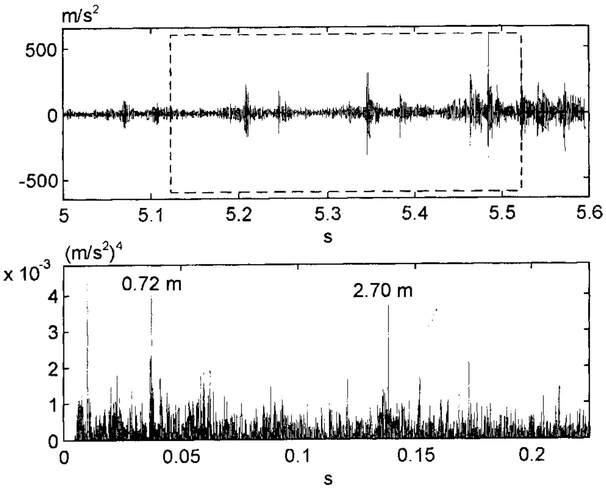

Fig. 6 First segment of acceleration time history of Interregionale train (Fig. 5) where cepstrum threshold is exceeded. Acceleration (top) and power cepstrum (bottom); equivalent distances are indicated for the main peaks

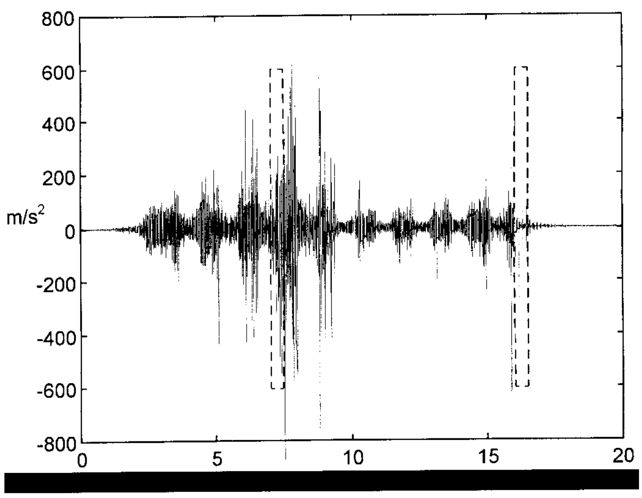

Fig. 5 Acceleration measured by accelerometer 2 for Interregionale train. Dotted lines indicate regions where cepstrum maximum exceeds the defined threshold

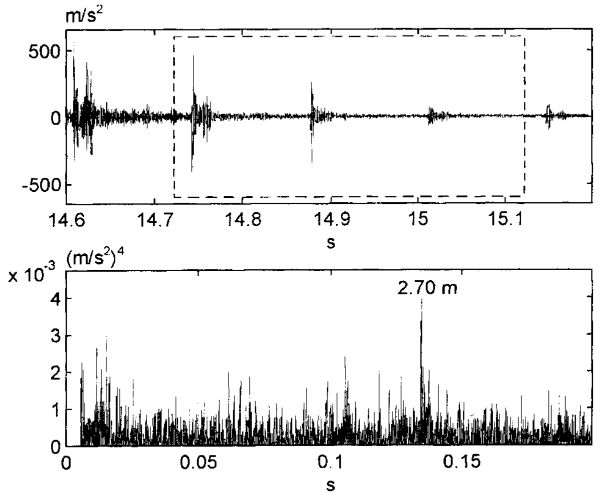

Fig. 7 Second segment of acceleration time history of Interregionale train (Fig. 5) where cepstrum threshold is exceeded. Acceleration (top) and power cepstrum (bottom); equivalent distances are indicated for the main peaks

listener automatically filters unwanted components and adjusts his threshold in real time.

The last bogie of the Interregionale train runs under similar conditions (Fig. 5). While recalculated energy levels are low, and this would make it still harder to find a defect, peaks are evident at regular intervals, expanding the zone that cepstrum indicates as potentially defective (Fig. 7). The cepstrum for this segment shows a single peak at the equivalent distance of 2.7 m, corresponding to a wheel circumference. This wheel has the tread in globally good condition with a wheelflat. It is important to underline that the recalculation of the energy for longer segments does not conflict with the first energy criterion: the latter (Fig. 2) shows that the bogie probably has some defects, while the former (Figs 3 and 4) firstly says that the wheel is not globally defective and secondly correctly locates the defect through the cepstrum analysis. The authors admit that this case is particularly lucky, as the defective bogie is the last of the train and the signal is detectable for longer, both by instruments and by the human ear, just as a result of the absence of other following bogies.

The case of globally worn tread is a trivial subcase and consists of high energy levels and uniform and very low cepstrum levels. Simple energy criteria are capable of finding only this kind of defect.

5 CONCLUSIONS AND FURTHER DEVELOPMENTS

The analysis of accelerations measured on the rail during the passage of trains in order to detect various types of defect in railway wheel treads has provided reliable results. Two signal processing techniques have been optimized to detect and classify wheel defects: energy analysis allows the identification of high excitations due to wheel corrugation, while cepstrum analysis can be used to relate acceleration peaks to wheelflats.

A crucial point still to be developed is the definition, for both the analysis methods, of statistically reliable thresholds. The best location for an automatic detector appears to be in the vicinity of terminal stations as:

(a) the speed lies in the optimal range (40–100 km/h) where the wheelflats effect is maximum;
(b) the speed is almost the same for all kinds of trains, and this makes the definition of thresholds easier;
(c) the vicinity of a station is certainly favourable for normal operation and maintenance; and
(d) requirements for the acquisition and signal processing devices are certainly lower than for similar applications on high-speed lines.

Wheel tread defects can lead to substantial damage to the track, to the vehicles and to the environment, dramatically increasing maintenance costs and decreasing intrinsic railway safety. It therefore appears very desirable to install automatic detectors that can identify and quantify these defects in their early stages, in a similar manner to what is achieved, for example, by using hot bearings detectors. As results obtained are particularly promising, methodologies used in this study will be refined by planning further measurement campaigns to optimize signal processing details and to test other kinds of transducers. At the same time, some parameters that have not been considered here, for example the great variation of wheel diameter and the strong influence of velocity on peak amplitudes, will be taken into account to design an automatic wheelflat and corrugation detection device.

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REFERENCES

1 Development of an acoustic device for the detection of wheelflats of a certain size. Office des Recherche et Essais of the Unione Internationale des Chemins de Fer, O. R. E. B110/ Rp.1/E, Utrecht, 1968.

2 **Ver, I. L.**, **Ventres, C. S.** and **Myles, M. M.** Wheel/ rail noise—part III: impact noise generation by wheel and rail discontinuities. *J. Sound Vibration*, 1976, **46** (3), 395–417.

3 **Ahlbeck, D. R.** An investigation of impact loads due to wheel flats and rail joints. Proceedings of ASME Winter Annual Meeting, Chicago, Illinois, 1980, pp. 1–10.

4 **Wiriyachai, A.**, **Chu, K.** and **Garg, V. K.** Bridge impact due to wheel and track irregularities. *J. Engng Mech. Div.*, 1982, **108**, (EM4), 648–667.

5 **Kobayashi, M.** and **Naito, T.** Vibration and noise of model wheel due to tread roughness. *Quarterly Reports*, 1983, **24** (3), 131–132.

6 **Raymond, G. P.** Track support must be right if concrete sleepers are to survive. *Railway Gazette Int.*, July 1984, 528–530.

7 **Fukuda, H.**, **Sasama, H.** and **Tanabe, S.** Studies for the use of acoustic diagnosis in the railways. *Quarterly Reports*, 1986, **27** (4), 131–135.

8 **Williams, S.**, **Ahlbeck, D.** and **Harrison, H.** Railroad bearing performance under the wheel impact load environment. Proceedings of the ASME Winter Annual Meeting, Boston, Massachusetts, 1987, pp. 1–7.

9 **Kaku, J.** and **Yamashita, M.** Impact noise from railroads. *J. Sound Vibration*, 1988, **120** (2), 333–337.

10 **Kumagai, N.**, **Ishikawa, H.**, **Haga, K.**, **Kigawa, T.** and **Nagase, K.** Factors of wheel flats occurrence and preventive measurements. *Wear*, 1991, **144**, 277–287.

11 **Cascini, G.** Un modello per la ricostruzione del segnale

prodotto da sorgenti mobili ed applicazione alle vibrazioni di binari ferroviari. Degree thesis, Università degli Studi di Firenze, Italy, 1996.

12 **Bracciali, A.** and **Cascini, G.** Time domain model of the vertical dynamic of a railway track up to 5 kHz. *Vehicle System Dynamics* (accepted for publication).

13 **Bracciali, A.** and **Biagiotti, M.** A method for measuring the dynamic behaviour of the Rome–Florence high-speed track switch manoeuvring system. *Proc. Instn Mech. Engrs, Part F*, 1992, **206** (F2), 107–116.

14 **Bracciali, A., Ciuffi, L.** and **Ciuffi, R.** Metodo innovativo per la misura della rumorosità esterna dei convogli ferroviari. *Ingegneria Ferroviaria*, 1994, **6**, 317–332.

15 **Bracciali, A., Ciuffi, L., Ciuffi, R.** and **Rissone, P.** Continuous external train noise measurements through an on-board device. *Proc. Instn Mech. Engrs, Part F*, 1994, **208** (F1), 23–31.

16 **Bracciali, A., Ciuffi, L.** and **Ciuffi, R.** Calibration of an on-board noise measuring device by simultaneous measurement of trackside noise of three different wheelsets for the ETR500 FS train. *Proc. Instn Mech. Engrs, Part F*, 1997, **211** (F1), 41–49.

17 **Kaper, H. P.** Wheel corrugation on Netherlands Railways (NS): origin and effects of 'polygonization' in particular. *J. Sound Vibration*, 1988, **120** (2), 267–274.

18 Wheel/rail contact noise—an experimental comparison of various systems for measuring the rail roughness associated with train rolling noise. Office du Recherche et Essais of the Unione Internationale des Chemins de Fer, O. R. E. C163/ Rp.9, Utrecht, 1988.

19 **Bogert, B. P., Healy, M. J. R.** and **Tukey, J. W.** The quefrency analysis of time series for echoes: cepstrum, pseudo-auto-covariance, cross-cepstrum and saphe cracking. Proceedings of the Symposium on *Time Series Analysis*, 1963, pp. 209–243 (J. Wiley and Sons, New York).

20 **Randall, B. P.** *Frequency Analysis*, 1987 (Brüel and Kjær, Nærum, Denmark).

21 **Norton, M. P.** Fundamentals of Noise and Vibration Analysis for Engineers, 1992 (Cambridge University Press, New York).

