ELSEVIER

Applied Acoustics, Vol. 54, No. 1, pp. 59–70, 1998

Printed in Great Britain
PII: S0003-682X(97)00044-3 0003-682X/98/$19.00 + 0.00

Least Squares Estimation of Main Properties of Sound Absorbing Materials through Acoustical Measurements

Claudio Braccesi[a] and Andrea Bracciali[b]

[a]Istituto di Energetica, Università degli Studi di Perugia, 06125 Perugia, Italy
[b]Dipartimento di Meccanica e Tecnologie Industriali, Università degli Studi di Firenze, 50139 Firenze, Italy

(Received 4 November 1996; revised version received 12 May 1997; accepted 19 May 1997)

ABSTRACT

Acoustical treatments with porous materials are widely used to reduce reverberation properties of closed spaces and to increase the transmission loss properties of multilayered panels. The optimization in a broad frequency range of sound absorbing layers encapsulated in multilayered panels requires exact knowledge of the acoustical behavior of porous materials and cannot be made with usual global acoustical parameters valid only at the higher frequencies. In this work a simple methodology to estimate the main acoustical parameters of porous materials from very simple experimental tests is developed and validated on several conventional and innovative materials. It uses a least squares regression based on measured reflection coefficient values of specimens to compute reliable values for sound absorbing materials parameters such as flow resistivity and structure factor. © 1997 Published by Elsevier Science Ltd.

Keywords: Absorbing materials, acoustical properties, least squares estimation.

NOMENCLATURE

c adiabatic sound speed in the air ($\mathrm{m\,s^{-1}}$)
c_p sound speed in the porous material ($\mathrm{m\,s^{-1}}$)
c_x equivalent sound speed in the rigid or limp porous material ($\mathrm{m\,s^{-1}}$)
f frequency (Hz)

h frequency response function (–)
k wavenumber (m^{-1})
K_s structure factor (–)
p sound pressure (Pa)
R flow resistivity (rayls m^{-1})
r reflection coefficient (–)
s air/porous material coupling factor (rayls m^{-1})
v particle velocity ($m\,s^{-1}$)
x abscissa (m)
Y porosity of the material (–)
Z_0 characteristic impedance of the porous material (rayls)
ω angular frequency (s^{-1})
γ propagation constant (m^{-1})
ρ_f density of the fluid (air) ($kg\,m^{-3}$)
ρ_m density of the fibers ($kg\,m^{-3}$)
ρ_p equivalent air density inside the material ($kg\,m^{-3}$)
ρ_x equivalent density for the rigid or limp porous material ($kg\,m^{-3}$)

INTRODUCTION

Porous materials are a fundamental component of passive acoustic treatments in any civil and industrial application and the literature regarding their properties and relative test methods has been extensive for several decades.[1–4] Their behavior is modelled with approaches of different degrees of difficulty but their use and design is often based on a few practical rules that use global acoustical characteristics obtained with standard test methods. Unfortunately these characteristics are not completely satisfactory since they are relative, and therefore applicable to the test configuration only.

If a more precise estimation of the interaction of porous materials with other materials and with the environment is required, it is necessary to measure specific parameters, suitable for use with complex mathematical models, to predict the performances of a multilayered panel in terms of reflection coefficient and of transmission loss.

On the basis of a mathematical model able to describe the behavior of both limp and rigid porous materials, an experimental–numerical procedure that permits reliable estimation of specific parameters of porous materials has been developed and tested. These parameters, namely the flow resistivity and the structure factor, are of crucial importance for the estimation of the acoustical properties of porous materials. The procedure developed here has been validated by comparing the experimental results with simulations performed using empirical relationships for the usual porous materials, while the properties of some non-conventional new acoustical materials have been derived.

We emphasize that the literature cited here is only that essential for the comprehension of the procedure developed and shown in the paper, while in the majority of more recent papers the acoustical modelling of the behavior of porous material is reconducted to the models cited in this work except for a few complex models, that are moreover valid only for some materials, that would have been unusable in the least squares regression procedure.

ACOUSTICAL MODELING OF POROUS MATERIALS

Fundamental equations

A brief description of more commonly used models is necessary to introduce the present work. Many researchers investigated the acoustical properties of porous materials using different approaches and then obtained results with differing accuracy. For example, one of the more complex models has been proposed by Biot,[5] and it allows the description of the behavior of a porous material with numerous boundary conditions. The behavior is, however, described by a few parameters that globally characterize the porous material.

The approach we used considers a simpler model that takes into account only the most important global parameters, in particular, neglecting intrinsic elastic and damping characteristics of the porous materials. It is clear that such a model cannot fit the characteristics of any porous material, but it nonetheless has proven to be very accurate and reliable for fiber materials that can be classified as limp or rigid.

Theoretical bases for these materials have been developed by many authors[6–8] who described their behavior in similar ways. We have chosen the Zwikker and Kosten[8] description since it is the more suitable for use with the experimental procedure described hereinafter.

Some definitions are necessary to introduce the equations that describe the behavior of the sound propagation inside a porous material:

- $Y = 1 - \rho_m/\rho_f$: porosity of the material;
- $c_p = c/\alpha$: sound speed in the porous material for a given thermodynamic process;
- $\rho_p = Y\rho_f$: density of the air inside the material;
- $s = Y^2 R + j\omega\rho_p(K_s - 1)$: air/porous material coupling factor.

While the porosity and the density of the porous material can be measured easily, the definition of velocity sound speed c_p needs a hypothesis about the thermodynamic process that the fluid undergoes inside the porous material. The sound speed in the air depends on the thermodynamic process that the fluid undergoes; if c is the adiabatic sound speed in the air, c_p depends on the fluid behavior inside the porous material. It can be proven that the ratio between the sound speed for adiabatic and isothermal processes is $\alpha = (1.41)^{0.5}$. For porous materials, the thermodynamic process is similar to an isothermal process, while at higher frequencies the process tends to become adiabatic. Moreover, for some materials whose fibers strongly assimilate to the motion of the fluid, the equivalent density can even be much greater than one (while the relation $\rho_p = Y\rho_f$ clearly makes $\rho_p < \rho_f$). In these cases c_p is completely different from both the isothermal and the adiabatic sound speed. Both α and ρ_p could have been included in the convergence procedure that will be discussed later, but this could have led to an increase in computation times and to a possible loss of stability of the algorithm. To offer the designer a chance to vary these two parameters, they are indicated explicitly in this work. A common hypothesis, used here, is that the process is isothermal, i.e. $\alpha = (1.41)^{0.5}$.

With these definitions, the interaction between porous material and air leads to the well known equations that enable the estimation of the pressure and velocity fields of the air inside the porous material for the propagation of a monodimensional acoustic wave of angular frequency ω:

$$\frac{\partial^2 \overline{p}}{\partial x^2} + \left(\frac{\omega}{c_x}\right)^2 \overline{p} = 0; \quad \frac{\partial v}{\partial x} = -j\frac{\omega}{\rho_x c_x^2}\overline{p} \tag{1}$$

These equations describes the propagation of the sound waves in a porous medium by considering a 'fictitious' fluid of equivalent characteristics c_x and ρ_x:

$$c_x = c_{\mathrm{RIGID}} = \frac{c_p}{\sqrt{1 - j\frac{s}{\rho_p \omega}}}; \quad c_x = c_{\mathrm{LIMP}} = c_p\sqrt{\frac{1 - j\frac{s}{\rho_m \omega}}{1 - j\frac{s}{\rho_p \omega}\left(1 + \frac{\rho_p}{\rho_m}\right)}} \tag{2}$$

$$\rho_x = \frac{\rho_p c_p^2}{c_x^2} \tag{3}$$

The equivalent properties c_x and ρ_x, different for rigid and limp material, are functions of the air/porous material coupling factor s that depends on the values of the flow resistivity R and the structure factor K_s. The flow resistivity R is defined as the ratio between the pressure drop in a unitary thickness of porous material and the velocity of the air flow, and represents the term that is responsible for the losses in the air/material coupling even if, as is clear by observing the definition of the coupling factor s, the structure factor also introduces a dissipation; the structure factor K_s summarizes all the other losses inside the porous material that are not strictly dependent on the flow resistivity R. These two parameters, with the velocity c_p, the porosity Y and the density ρ_m, allow the application of analytical models to describe the acoustical behavior of the porous material.

Other definitions with the same physical implications, for example the *tortuosity*, that can be found in the literature are equally valid. It is well known that many parameters that characterize the acoustical behavior of porous materials can be determined with great accuracy using non-acoustical test methods, but this clearly requires specific test devices that were not available for the present research, while one of the goals of the work was to prove that the developed method can be reasonably applied with only basic acoustical transducers and setup. The results shown in this paper appear to be accurate enough to provide useful engineering models for the industrial designer, and for the porous materials manufacturer, with the least time and costs. The use of other methods (for example the use of the impedance tube with the two thicknesses method or with the conceptually equivalent two impedances method) can give, in a straightforward manner, the characteristic impedance Z_0 and the propagation constant b, quantities that depend on the chosen phenomenological parameters, knowledge of which is fundamental to identify the instrinsic properties of the materials and then to improve the choice of materials and the design process. A least squares procedure, similar to the one shown here, could have been developed starting from Z_0 and b to find the parameters K_s and R that are considered to be the most useful for the aforementioned reasons. Moreover, the two thicknesses method cannot be applied to materials manufactured with one thickness only, or when the material is not available for arbitrary thicknesses with the same specific properties; the use of two specimens can lead to incorrect results for the indetermination of the interface impedance.

Matrix formulation for porous materials

Figure 1 shows a plane sound wave incident on a porous material of thickness L with an angle ϑ. The solutions of eqn (1) relate air pressures and velocities upstream (A subscript) and downstream (B subscript) of the porous material with the matrix

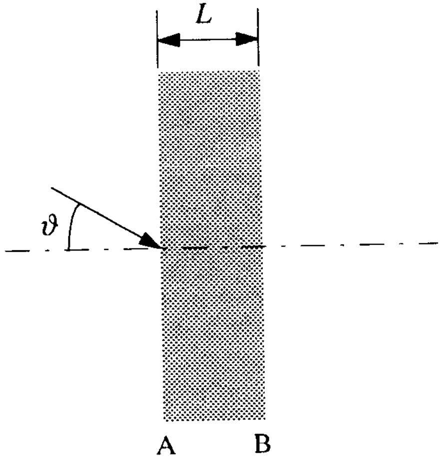

Fig. 1. Sketch of the geometry for a plane sound wave acting on a porous material sheet of thickness L.

$$\begin{Bmatrix} p \\ v \end{Bmatrix}_A = \begin{bmatrix} \cosh(\gamma L) & Z\sinh(\gamma L) \\ \dfrac{\sinh(L)}{Z} & \cosh(\gamma L) \end{bmatrix} \begin{Bmatrix} p \\ v \end{Bmatrix}_B \tag{4}$$

where

$$k = \omega/c;\ \ \gamma_0 = j\omega/c_x;\ \ \gamma = \gamma_0\sqrt{1 + \left(\frac{k}{\gamma_0}\right)^2 \sin^2\vartheta};\ \ Z_0 = \rho_x c_x;\ \ Z = \gamma_0 Z_0/\gamma \tag{5}$$

Matrix formulation easily allows the simulation of multilayered panels of various materials for which equations similar to eqn (4) can be written. We observe that an air layer can still be described by eqns (4) and (5) assuming $\gamma = \gamma_0 = jk$ and $Z = Z_0 = \rho_c$. The simulated characteristics of limp and rigid porous materials obtained using these equations are shown in eqn (2), where the normalized propagation constant γ/k and acoustic impedance Z_0/ρ_c for normal incidence are reported for limp and rigid porous materials as a function of the frequency, highlighting the differences of these properties for porous materials with respect to the air, for which $Z_0/\rho_c = 1$ and $\gamma/k = j$.

Generally speaking, porous materials have a complex propagation constant γ_0 and acoustic impedance Z_0. The real part of the propagation constant is directly related to sound wave attenuation inside the porous material; the acoustic impedance is high at low frequencies and asymptotically tends to that of the air at higher frequencies. Figure 2 also shows that limp and rigid porous materials differ only at lower frequencies.

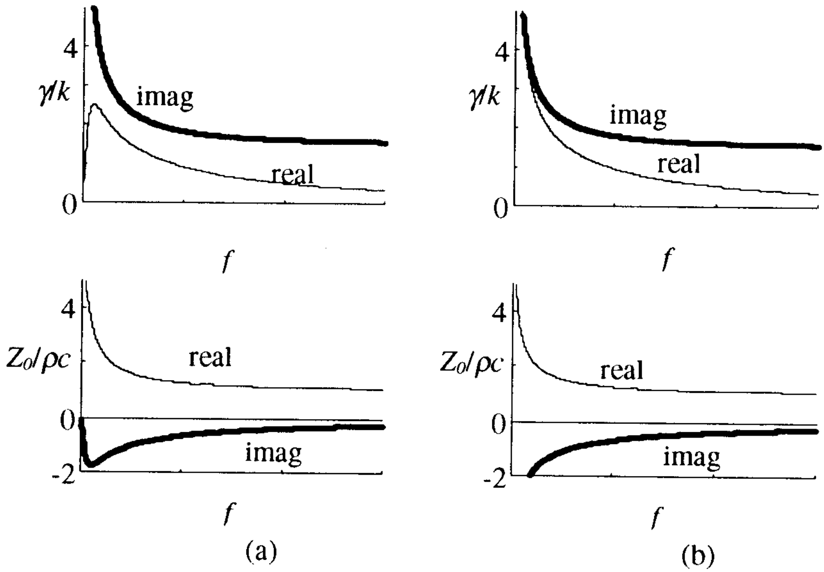

Fig. 2. Typical specific propagation constant and specific impedance for (a) limp and (b) rigid porous materials.

ACOUSTICAL CHARACTERIZATION OF POROUS MATERIALS

The most important parameters used to characterize a porous material are the flow resistivity R and the structure factor K_s. Since direct measurement of R and K_s is very difficult, an indirect estimation of these parameters can be made through an experimental procedure where only the reflection coefficient of the porous material is measured.

If a specimen of thickness L of porous material is subjected to a normal incident sound wave ($\vartheta = 0$) and the downstream section is coincident with a rigid termination (i.e. $v_B = 0$), it is easy to find from eqn (4) that the acoustical impedance at the front surface of the porous material is $Z_A = p_A/v_A = Z_0\coth(\gamma_0 L)$. It follows that the reflection coefficient of the front surface of the porous material is

$$r = \frac{Z_A - \rho_c}{Z_A + \rho_c} \qquad (6)$$

that is a complex quantity that globally characterize the porous material and can be computed from given R and K_s.

The reflection coefficient has been measured using the test rig shown in Fig. 3, that has been designed and verified following the norm ASTM 1050-86[9] to guarantee the reliability of the results in the frequency range of

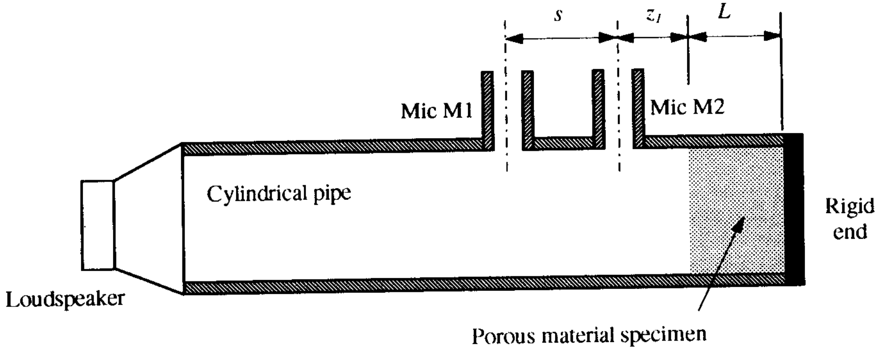

Fig. 3. Experimental setup.[9]

interest. A specimen of the porous material of thickness L has been placed at the rigid closed end of a straight cylindrical pipe in which a loudspeaker introduces plane sound waves. The loudspeaker is driven by an amplified random noise signal, and two flush mounted measurement microphones M1 and M2 measure the sound pressure in two sections of the pipe at a distance s. Defining as $h = p_2/p_1$ the measured frequency response function between the two microphones, it can be proven that the experimental reflection coefficient is given by

$$r = \frac{e^{jk2z_1}(e^{jks} - h)}{h - e^{-jks}} \tag{7}$$

Equations (6) and (7) allow the direct comparison of measured and estimated results, obviously if the acoustical parameters R and K_s of the material are already known.

For materials not already characterized, the estimation of R and K_s has been made through a numerical procedure of convergence of the numerical results of eqn (6) on the experimental results of eqn (7) with a least squares criterion, therefore automatically varying the flow resistivity R and the structure factor K_s until a good correlation is reached. The calculated parameters can be used with good reliability in a mathematical model that simulates the behavior of multilayered panels with one or more layers made of that particular porous material.

EXPERIMENTAL RESULTS

To validate the experimental setup and the least squares iterative estimation procedure, tests were initially performed on a fiberglass porous material for

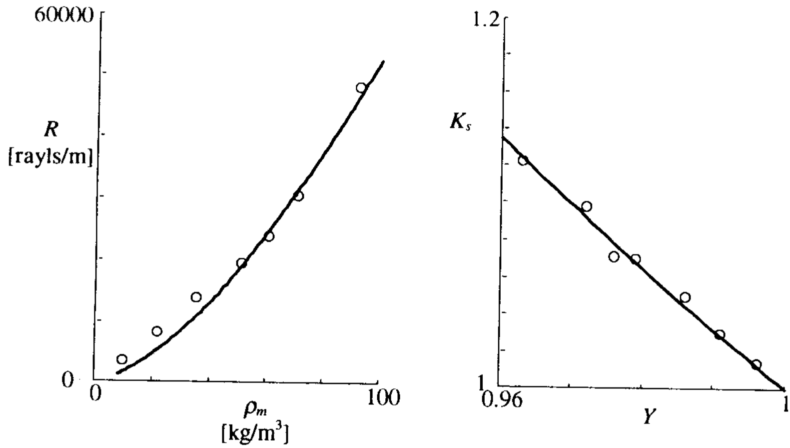

Fig. 4. Flow resistivity R and structure factor K_s for all tested fiberglass specimens. ○, experimental values, lines: from Beranek's[6] laws.

which plenty of data are available in the literature. Several fiberglass specimens of different density ρ_m have been selected, while the average fiber diameter d has been measured with a microscope. Between the numerous semiempirical relationships available in the literature, the chosen one is[6]

$$\frac{Rd^2}{\rho_m^{1.53}} = \text{const}; \quad K_s = Y^{-3.1} \tag{8}$$

that relates the flow resistivity with the average fiber diameter and the structure factor with the porosity. Figure 4 allows the direct comparison of experimental results with the values calculated with eqn (8). The agreement is

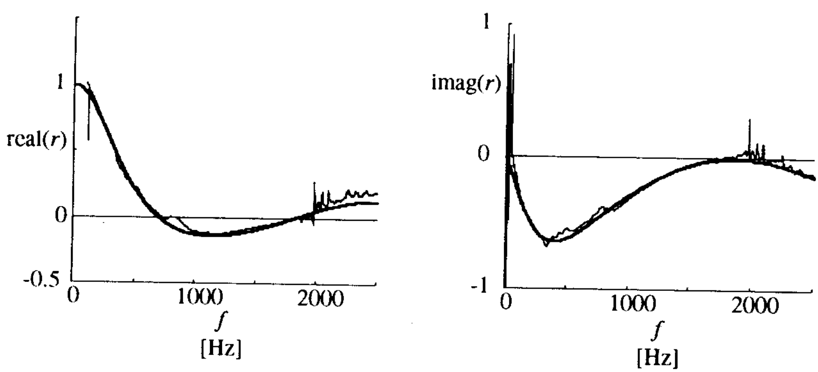

Fig. 5. Real and imaginary part of the reflection coefficient for a fiberglass specimen with $\rho_m = 51.2\,\mathrm{kg\,m^{-3}}$. Converged parameters: $R = 19\,500\,\mathrm{rayls\,m^{-1}}$, $K_s = 1.07$. Thick lines, calculated after least squares process; thin lines, experimental.

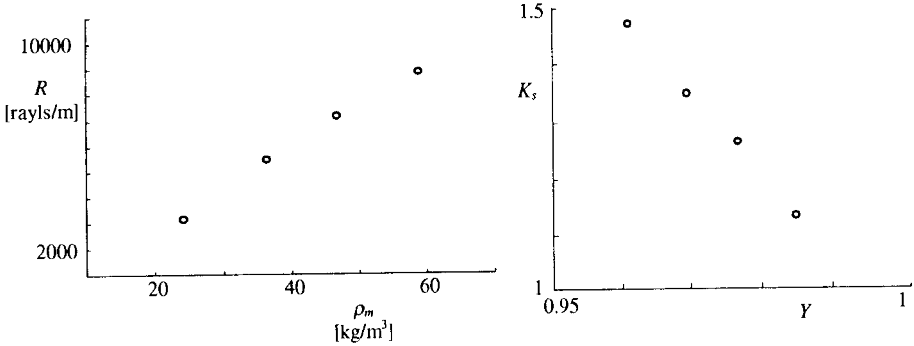

Fig. 6. Experimental values of flow resistivity R and structure factor K_s for all tested compacted polyester fiber specimens.

very good, and Fig. 5 shows the real and imaginary parts of the experimental reflection coefficient and the same curves obtained numerically after the least squares convergence process used to estimate R and K_s.

A material made of polyester fibers of different densities and specially treated to obtain waterproof and fire autoextinguishing characteristics has been tested. For all the specimens the fiber material has a density $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$ with an average fiber diameter of 37 μm. Figures 6 and 7 show the results obtained after the least squares estimation process completion for flow resistivity and structure factor. For this material eqn (8) proved not to be valid and the estimated structure factor is much greater than that predicted, probably due to the chemical components used to compact the base material that partially fill the pores, and increase the acoustical dissipations not directly related to flow resistivity. No attempt has been made to derive a rule similar to eqn (8) for this material, as the number of commercially

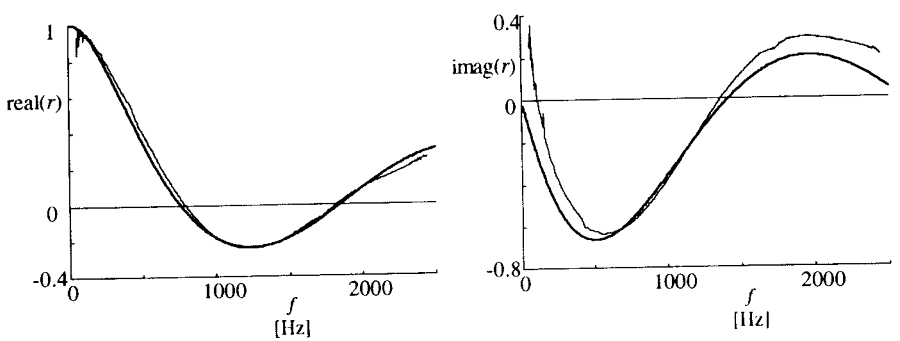

Fig. 7. Real and imaginary part of the reflection coefficient for a polyester fiber with $\rho_m = 46.7\,\mathrm{kg\,m^{-3}}$. Converged parameters: $R = 7200\,\mathrm{rayls\,m^{-1}}$, $K_s = 1.32$. Thick lines, calculated after least squares process; thin lines, experimental.

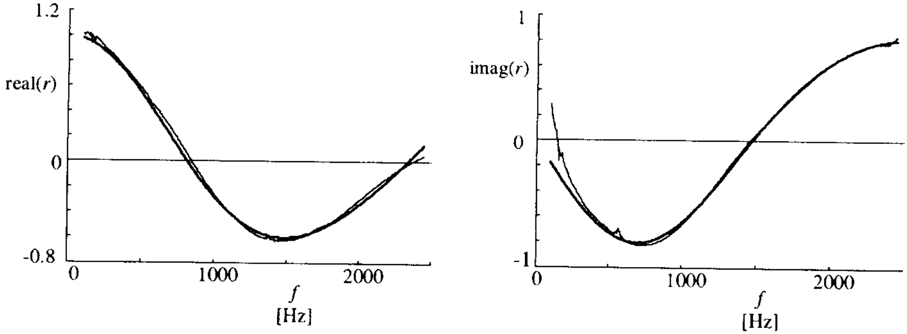

Fig. 8. Real and imaginary part of the reflection coefficient for a porous aluminium fiber with $L = 1.2$ mm. Converged parameters: $R = 105\,000$ rayls m^{-1}, $K_s = 9.67$. Thick lines, calculated after least squares process; thin line, experimental.

available fiber densities is very limited and moreover it lies beyond the scope of this work.

A new material, made of pressed aluminum fibers ('porous aluminum') has been tested. Commercially available thicknesses are in the range 0.8–1.6 mm and since they are too thin to be efficiently measured directly at the rigid end, all measured specimens have been placed at a fixed distance (50 mm) from the end to allow the sound wave to cross them. The reflection coefficient now depends on the characteristics of the material placed in series with an air layer that ends on a rigid termination. The least squares estimation process can still be applied to deduce the parameters R and K_s of the unknown material, since air characteristics are well known. Figure 8 shows the convergence obtained for a 1.2 mm thickness specimen with 50 mm air behind it. The value of the structure factor K_s seems quite high; no theoretical investigations have been performed, but this relatively new material has properties different from classical porous materials; however, the usual model for porous materials applied to experimental results seems to fit the acoustical response of this material.

CONCLUSIONS

In this paper a simple methodology to estimate acoustical characteristics of porous materials has been proposed and tested. It uses reflection coefficient data to estimate the flow resistivity and the structure factor using a least squares fitting procedure. It gives results that are in good agreement with the literature results for known materials, and allows a fast and reliable estimation for new porous materials or for materials that do not follow common empirical laws.

REFERENCES

1. Goransson, P., A weighted residual formulation of the acoustic propagation through a flexible porous material. *Journal of Sound and Vibration*, 1995, **182**, 479–494.
2. Ookura, K. and Saito, Y., Transmission loss of multiple panels containing sound absorbing materials in a random incidence field. Proceedings of Inter-noise 1978, pp. 637–642.
3. Biot, M. A., The theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low frequency range, II. High frequency range. *Journal of the Acoustical Society of America*, 1956, **28**, 168–191.
4. Nichols, C. A. Jr., Flow resistance characteristics of fibrous acoustical materials. *Journal of the Acoustical Society of America*, 1947, **19**, 866–871.
5. Biot, M. A., Generalized theory of acoustic propagation in porous dissipative media. *Journal of the Acoustical Society of America*, 1962, **34**, 1254–1264.
6. Beranek, L. L., *Noise and Vibration Control*. McGraw-Hill, New York, 1971.
7. Ingard, K. U., Locally and non locally reacting flexible porous layer: a comparison of acoustical properties. *ASME Transactions, Journal of Engineering for Industry*, 1981, **103**, 302–313.
8. Zwikker, C. and Kosten, C. W., *Sound Absorbing Materials*. Elsevier, Oxford, 1949.
9. American Society for Testing Materials, Standard test method for impedance and absorption of acoustical materials using a tube, two microphones and a digital frequency analysis system, norm ASTM E, 1050–1086.