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HIGH-FREQUENCY MOBILE INPUT RECONSTRUCTION ALGORITHM (HF-MIRA) APPLIED TO FORCES ACTING ON A DAMPED LINEAR MECHANICAL SYSTEM

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The direct measurement of mobile input forces in the high-frequency range is often impossible in many mechanical systems. In this paper, an algorithm for the reconstruction of a mobile input based on a parametric model of a system and on its response to the unknown input is proposed and validated. To apply the algorithm, the system must be described in terms of models of discrete poles and zeros. A simple damped mechanical system with complex behaviour (constrained layer damped dual beam) identified with an ARMAX (autoregressive moving average with external input) model was used in this study. The output signals used in the algorithm validation phase were estimated with a custom procedure for narrow- and broadband mobile input forces.

1. INTRODUCTION

The direct measurement of high-frequency input forces is practically impossible in many complex mechanical systems. A classical example is the measurement of the force mutually exchanged by a wheel and the road or rail. In such cases, it is common to estimate contact forces indirectly through the measurement of the deformation of some mechanical devices (springs and suspension arms or axle and wheels) with strain gauges using, when appropriate, telemetry systems.

For static or quasi-static analysis, the (constant) relationship between the measured output deformation and the input unknown force can be found easily by a static calibration of the measurement chain supplying a set of known forces to the system.

Unfortunately these methods have limitations in the high-frequency range, typical, for example, of noise problems. In these cases the relationships between the input and the output become very complicated as these relationships are no longer a constant value or a matrix of constant values, but a set of complex transfer functions whose magnitude and phase are a function of the frequency. Moreover, strain gauges, and their conditioning chain, cannot be used beyond an upper frequency limit which is too low to be used in high-frequency vibration detection, and the use of accelerometers with proper measuring range requires expensive and delicate telemetry systems.

Alternatively, the estimation of input forces can be made through indirect signal reconstruction techniques using the third law of the dynamics; for example, by measuring accelerations in predetermined locations. These techniques process output signals (i.e. accelerations or sound pressure) with reconstruction algorithms that use mathematical models of the system.

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If the system over which the travelling force is moving is divided into a certain number of finite dimension elements, some peculiar problems arise for the reconstruction of the mobile force. If the system is modeled as a multiple input multiple output (MIMO) one, the number of inputs is too large to be reconstructed efficiently with any inversion or reconstruction procedure. The fact that the force acts just over one point—or section—while it is zero over the others, should be properly taken into account to simplify any inversion/reconstruction procedure; for example, by considering only the instantaneous equivalent single input single output (SISO) system that relates the force acting in the actual section and the corresponding output signal.

The approach using the frequency response functions (FRFs) of the system and the spectral properties of input and output signals is limited by the reduced number of lines (not—except for particular cases—a power of 2) that leads to a definition of the signals in the frequency domain which is too coarse even for slow-moving forces. In the most general case of varying speed, there is a continuous variation of the number of frequency lines used to represent the input/output signals and consequently the number of lines of the FRFs. Moreover the output in a given location at the instant t^* is nothing but the sum of the contributions of the force that passed over previous sections for $t < t^*$ (i.e. the length run by the mobile force); the windowing treatment of the signal before Fourier transform, to reduce the well-known leakage problems, introduces inevitable distortions in the signals that have to be superimposed.

In this paper, a high-frequency mobile input reconstruction algorithm (HF-MIRA) is proposed and validated for a system with transfer functions strongly variable with distance. While this work focuses on the reconstruction of a mobile force acting on a complex mechanical system from acceleration measurements, it can be applied to any linear system (electrical, mechanical, etc.) sufficiently damped where simpler system modeling techniques fail.

HF-MIRA is applied to a system modeled from input and output measured time histories, since classical lumped-parameter models are not suitable for applications in the high-frequency range. This modeling approach, typical of electronic systems analysis, provides parameters that are not related directly to the general properties of the kind of structure, but are applicable only to the measured one. The good description at higher frequencies is therefore possible only with a loss of generality and of correlation to the physical properties of the structure.

The model proposed here is intrinsically very different from more common input reconstruction approaches based on frequency analysis, and a direct comparison is therefore possible only in terms of obtained results. The features of the approach, mainly the possibility to use short sequences without windows or fast Fourier transform (FFT) limitations, make it very attractive for use with arbitrary length signals with the only indetermination of the head and tail sequences for a number of samples related to the number of parameters (poles and zeros) of the autoregressive model of the structure.

The validation of the HF-MIRA should be made through the comparison between measured and reconstructed input forces given by a mobile source. Since direct measurement of mobile input forces in the high-frequency range is, as already mentioned, almost impossible, in this work the output has been estimated by a numerical application of a mobile excitation.

Therefore, the developed HF-MIRA is used in a procedure that consists of: (1) choice, characterisation and high-frequency modeling of a damped mechanical system; (2) numerical application of an arbitrary mobile input and estimation of the output without using the model of the structure; (3) input reconstruction with HF-MIRA and comparison with the supplied input.

2. SELECTION AND MODELING OF A COMPLEX MECHANICAL STRUCTURE

The HF-MIRA is particularly advantageous in the analysis of highly damped systems whose FRFs are relatively 'flat' and therefore can be described with a limited number of parameters. This way the behaviour in the whole frequency range can be described with a light model with good precision in terms of both amplitude and phase. The procedure shown here has been developed to be applied to signals collected on railway tracks, that exhibit a very damped behaviour (the track is in fact a waveguide and does not go to eigenmodes).

Due to the importance of the selection of a structure that is similar to a railway track, the choice and the modeling of a test structure are crucial for the validation of HF-MIRA.

2.1. SELECTION AND CHARACTERISATION OF A COMPLEX MECHANICAL STRUCTURE

The selected structure should be linear, observable, time invariant and should be sufficiently damped (FRF magnitude range of about 20–30 dB *vs* the normal dynamics of more than 60–70 dB of almost undamped structures) without any prevalent peak in the frequency response. Under these hypotheses, the structure can be divided into sections and described through a limited number of experimentally measured FRFs that can be considered constant for any input inside each section.

The chosen structure was supported on sufficiently compliant constraints to obtain the free–free behaviour (Fig. 1). It satisfied all the above-mentioned hypotheses, and its characterisation, necessary for the construction of the model, was made by using its dynamical response to measured input forces. The structure was divided into 13 sections of equal length; for any section the response was considered invariant independently of the point where the force was applied inside the section. The test forces were supplied in the midpoint of each section and the output was measured only in the first section (point 1).

An instrumented hammer (Bruel&Kjaer 8001) was used to excite the beam. The stiffnesses of the tip of the hammer and of the structure were such that the contact force had sufficient frequency content up to 10 kHz. These conditions were very similar to those normally encountered on solid steel bodies, and therefore will be applicable also during railway track testing where the use of an electrodynamic shaker, absolutely equivalent for the goal of this work, would prove to be unfeasible for safety and traffic regularity reasons. Output accelerations were measured with a 10-g monoaxial piezoelectric accelerometer fixed with cyanoacrilate, a solution that ensures the maximum mounted natural frequency of the transducer.

The estimation of FRFs was performed by weighting time force signals with a truncated exponential window and by weighting acceleration signals with an exponential window. This procedure does not introduce magnitude and phase errors in the estimation of the FRF as they are automatically compensated by the algebraic operations (division). Six measurements were performed in each section with a 25.6-kHz sampling frequency. Each

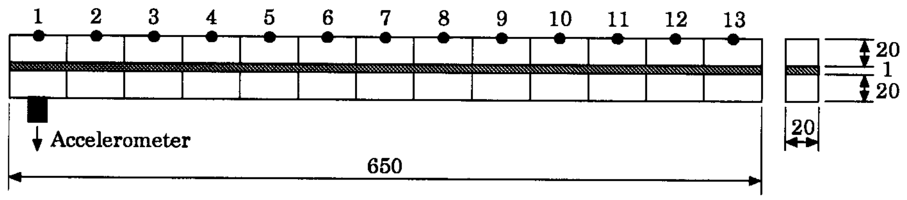

Figure 1. The chosen mechanical structure with output and input locations used for the characterisation. The structure is composed of two steel cross-squared beams with a glued viscoelastic damping layer. ●, Excitation points.

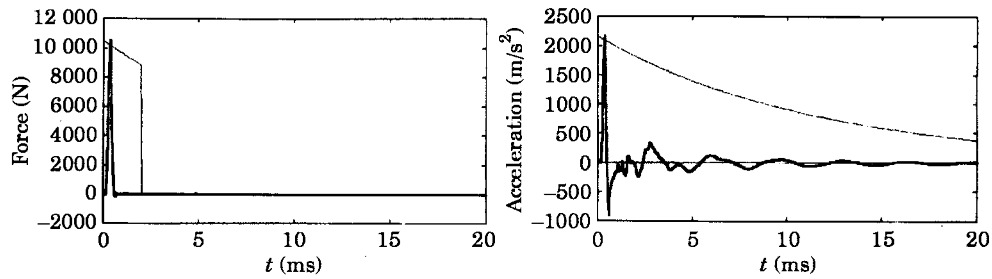

Figure 2. Time history of a hammer blow in the excitation point 1 and of the accelerometer signal in the same section (——) and time windows used for weighting (——). Time is limited to 512 samples, while the complete acquisition lasts 2048 samples.

acquisition contained 2048 samples, more than sufficient for the decay of free vibrations (Fig. 2). FRFs were computed with the estimator H_1 that minimises random errors in the antiresonances (Fig. 3). The capability of the front-end to estimate H_1 directly was not used since raw time histories are necessary later for the system identification. It is worth noting that the FRFs were used in this work only for output estimation (a step that would not exist if the input force were actually unknown) and for pictorial comparison between the model and real behaviour of the structure.

2.2. MODELING THE SELECTED STRUCTURE

Mechanical systems are usually modeled with lumped-parameter models (mass–damper–spring) or with non-parametric models [through FRFs or input response functions (IRFs)].

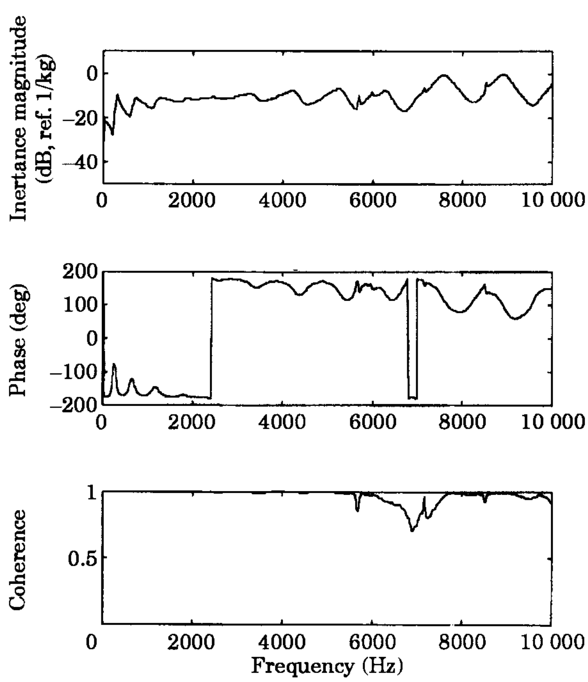

Figure 3. Magnitude, phase and coherence (six averages) of point FRF $H_{1,1}$. The frequency content of both force and acceleration signal is particular high in the whole range, as highlighted by the very good value of the coherence function.

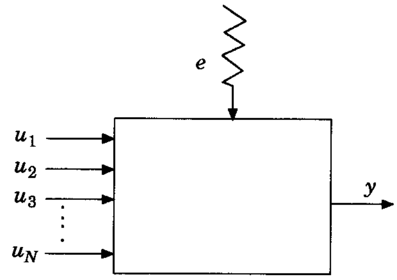

Figure 4. Block diagram of a MISO system in presence of extraneous noise or unmeasured input.

The first method, i.e. the modal approach, gives parameters that are related directly to the physical properties of the system, but it is not applicable to highly damped systems with coupled modes, high modal density or local modes. Some authors have tried to apply it to railway tracks [1], but its validity at higher frequencies remains questionable. The second method is absolutely general and is particularly efficient in modeling the output in the measured degrees of freedom due to any input. This method has severe limitations when applied to input reconstruction techniques for more complex systems [2] since the procedure requires the inversion of strongly ill-conditioned matrices.

The model used here, based on the processing of the input/output time-history pairs collected during the characterisation phase, has none of the limitations of the above methods. It is applicable to any linear structure and does not lead to any matrix inversion in the input reconstruction procedure. In the following, the time-discrete signals are indicated generically with $x(t)$, where $t = 0, \Delta T, 2\Delta T, \ldots$ where ΔT is the sampling period. The model of the structure has the following fundamental characteristics (for further details see [3, 4]).

- It is parametric and describes the behaviour of the system through a finite number of parameters that are not directly related to the physical properties of the system but only depend on how input signals are 'transferred' to the output.
- Parameters are extracted from superabundant information, with overdetermined computations. Parameter estimation results improved; it is also possible to estimate the indetermination of the parameters and, with appropriate test techniques, to reduce it.
- It is possible to estimate the effect of noise or of unmeasured inputs on the output of the system. If this effect is small the output depends only on the measured inputs and it is possible to estimate the inputs from the output signals, otherwise some properties that depend on the noise will be attributed to the input.
- As the number of parameters is relatively small, the global behaviour of the model is more stable than with the models based on FRFs where it is possible by no means to ensure that the coherence (and hence magnitude and phase) of FRFs is good in any frequency line. Since a substantial contribution to the reconstructed force is given by antiresonances in the FRFs (that must be used inverted), a small error in the measurements, particularly high at the antiresonances for signal-to-noise ratio reasons, leads to large errors in the estimated force. Parametric models instead smooth experimental FRFs, reducing 'single-line' errors (Fig. 5).

The system has been identified with the ARMAX model shown in Fig. 4, which is sufficiently accurate to model the effects of uncontrolled inputs on the output. The characteristic equation of an ARMAX model for a SISO system is

$$A(q)y(t) = B(q)u(t - nk) + C(q)e(t) \tag{1}$$

where q, called shift operator, is such that

$$q\, u(t) = u(t+1)$$

and $A(q)$, $B(q)$, $C(q)$ are polynomials of delay operator q^{-1} in the form

$$A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{na} q^{-na}$$

$$B(q) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{nb} q^{-nb}$$

$$C(q) = 1 + c_1 q^{-1} + c_2 q^{-2} + \cdots + c_{nc} q^{-nc}$$

and nk is the number of delays between the input and the output, namely the minimum number of sampling intervals ΔT needed to 'transmit' the input signal to the output.

For mobile inputs, with the cautions and under the hypotheses described previously, it is always possible to consider SISO systems as the mobile force acts separately on each section, and equation (1) can be rewritten as:

$$\begin{aligned} y(t) &+ a_1 y(t-1) + \cdots + a_{na} y(t-na) \\ &= b_1 u(t-nk) + b_2 u(t-nk-1) + \cdots + b_{nb} u(t-nk-nb+1) \\ &+ e(t) + c_1 e(t-1) + \cdots + c_{nc} e(t-nc). \end{aligned} \tag{2}$$

For a general MIMO system, the equation is formally equal to equation (1) where a_i and c_i coefficients are matrices $ny \times ny$, b_i coefficients are matrices $ny \times nu$, and nu and ny are respectively the number of the inputs and of the outputs.

The system has been modeled by using the libraries of the Matlab® Identification Toolbox [5]. Parameter estimation is performed with the 'prediction error method' that iteratively reduces the least squares error made in the representation of the behaviour of the system. First, it is necessary to declare the number of parameters that describe the system, i.e. the dimensions of A, B and C, and the delay for each input. Note that the modelisation of mechanical systems usually requires more parameters than for electronical systems, due to the higher complexity of FRFs curves.

To build the model correctly it is necessary to give to the parameter estimation algorithm the most general time domain force–acceleration signal pairs, such that all the needed information is supplied properly. If the nature of the source can be estimated, it is better to build the model from signals that have properties (amplitude and frequency distribution) similar to the real ones. If the amplitude of the test signals is similar to those measured during real excitation, small non-linearity problems are minimised.

Hammer blows and relative responses are suitable for this, as they cover all the investigated frequency range. Moreover, they simulate the wheel–rail excitation due to the contact roughness. In this paper, no measurements of acceleration under real loads are available and so no constraints exist on the amplitude of input funcions used for characterisation, as the structure proved to be linear for a wide range of input force amplitudes.

The structure was modeled with 36 poles for the only measured output and 35 zeros for each input. The number of zeros did not need to be equal for all the inputs, but this choice proved to be very efficient for processing reasons. The number of delays varied from one to four depending on the distance between the input and output sections used in characterisation, as wave propagation velocity in steel is not infinite (about 5 km/s). The Doppler effect is neglected as the velocity of the travelling force is much less than sound speed in the structure. The number of parameters seems very high, and could probably be reduced slightly, but it must remain of the same order to faithfully 'copy' the FRFs.

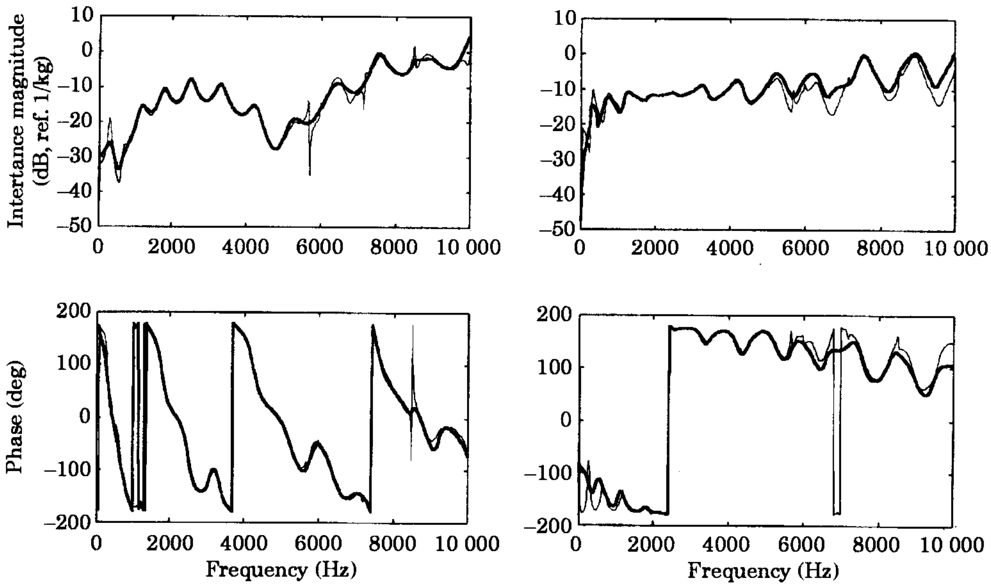

Figure 5. Magnitude and phase of transfer $H_{1,12}$ (left) and point $H_{1,1}$ (right) FRFs. ——, Measured; ——, reconstructed with ARMAX model.

Figure 5 shows a transfer and the point FRFs that prove that the model has very good FRFs reconstruction capabilities in the whole frequency range, both in amplitude and phase. The position on the z-plane of the poles and the zeros for input 12 is shown in Fig. 6 together with the unit circle. The almost uniform distribution of poles and zeros confirms the need for an accurate modeling of the system over the whole frequency range.

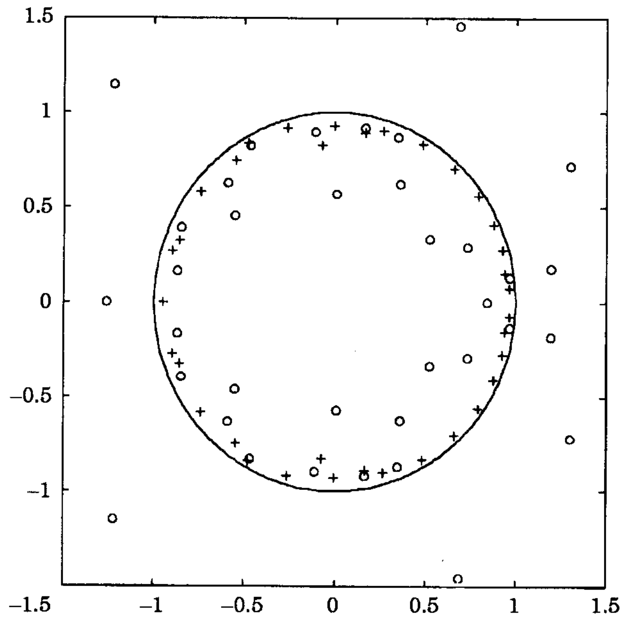

Figure 6. Poles (+) and zeros (○) of FRF $H_{1,12}$ on the z-plane.

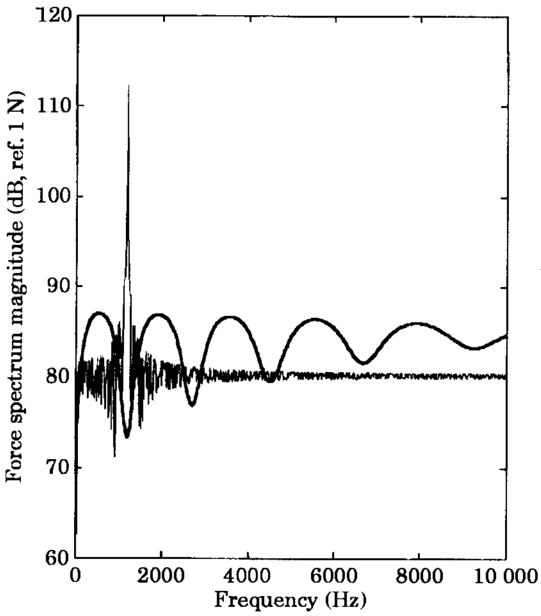

Figure 7. Magnitude of narrow- and broadband input signals applied to the system. Phases (not shown) are random. The energy of the two signals is similar.

3. SIMULATION OF THE APPLICATION OF A MOBILE INPUT SIGNAL

For a correct verification of the applicability of the model and of the reconstruction algorithm it is necessary to preliminary take into account that the force(s) used for input simulation must be as general as possible and that the numerical input must give an output that is as close as possible to the response of the system to the selected mobile force. This last precaution is fundamental since it is not possible to measure a real mobile excitation.

3.1. SELECTION OF THE MOBILE FORCE

To ensure the maximum generality of the proposed reconstruction procedure in the whole frequency range two complementary types of forces have been numerically applied to the system (Fig. 7): (1) a broadband force signal, with an approximately constant frequency distribution ($\cong$20 dB range) and random phase (similar to the one supposed for wheel–rail contact force [6]); and (2) a narrowband force signal, i.e. a pure sine superimposed to a -30-dB white noise. This signal is similar to real ones, in which higher order harmonics or broadband noise are normally found in measurements. Random phases are typical of real systems; an averaging process is therefore necessary to find energy properties of reconstructed signals.

3.2. NUMERICAL MOBILE FORCE APPLICATION AND OUTPUT ESTIMATION

If a random mobile force passes over the structure, each element in Fig. 1 is subjected to a slice of input signal uncoupled with the others and, if the system is linear, the output can be estimated by superimposing the responses due to the relative inputs. A square force distribution, however, seems to be too far from the real one, and a more plausible application can be obtained by distributing the signals on two consecutive sections in a triangular way. Supposing that the FRF varies linearly between two measurement points, the mobile force application process should take into account this condition. For software implementation reasons, FRFs are retained as constants, and the input force is distributed so that it acts completely on an element when over the midpoint of the element and linearly decreases as it moves to the midpoint of the following element (left of Fig. 8).

The response to each input is computed through the convolution of the input with experimental IRFs, obtaining the corresponding output under that input only. This procedure does not use the ARMAX model of the structure, thereby limiting numerical 'manipulations' of the signals. The global output is obtained by simply summing up all the outputs (right of Fig. 8). The example shown here is for a constant velocity mobile force, even if there are no limitations for arbitrary motion laws.

4. HF-MIRA DESCRIPTION

The estimation of input forces is theoretically possible, inverting the FRFs matrix, for systems with equal number of inputs and outputs ($nu = ny$). If $nu < ny$ (algebraically overdetermined), the information redundancy makes input estimations more robust, since the use of the pseudo-inverse matrix provides the least squares error solution. Unfortunately the FRF matrices to be inverted are often ill-conditioned, such that small measurement errors result in strong errors in estimated forces especially at the antiresonances. Ill-conditioning increases with the number of inputs, and if $nu > 3$ the redundancy of measured output has a low beneficial effect [2].

The developed HF-MIRA uses a parametric model of the structure and the output under the real mobile force. In this work the algorithm is verified through the use of the experimental ARMAX model of the structure (Section 2) and the simulated response (Section 3) to the mobile force to be reconstructed.

If the mobile force velocity is known, then the passing time on each section is known also. The system is apparently composed of a number of inputs equal to the number of sections, but since during each lapse the force acts only on one section, it is possible to

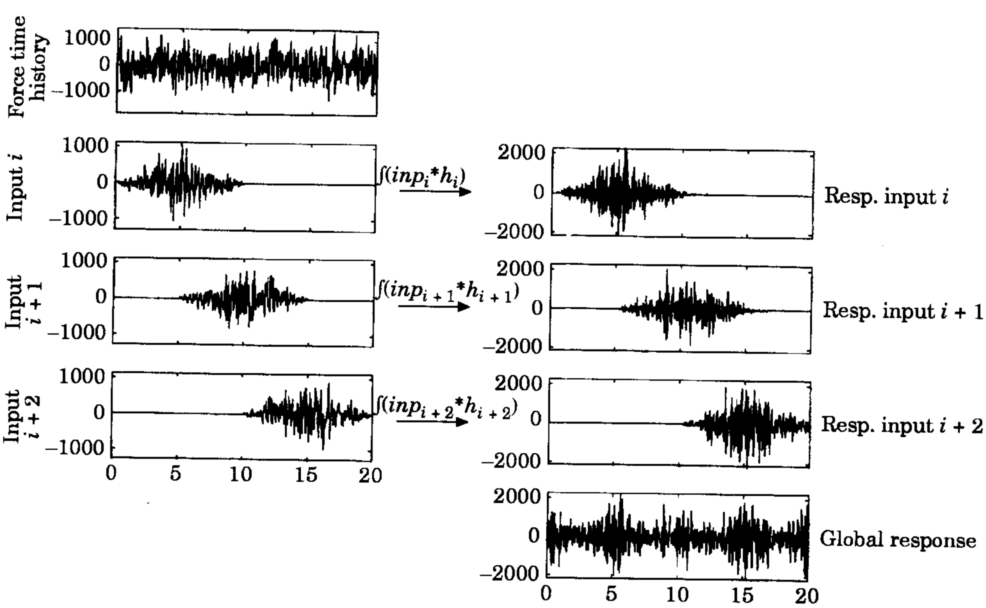

Figure 8. Application of the simulated narrowband mobile input signal to the system. The signal is distributed linearly on adjacent elements as explained in the text (left). Estimated output signals obtained through convolution with corresponding measured impulse response functions without using the ARMAX model of the system. The global response is the sum of each computed output (right).

consider the system as an equivalent SISO system that does not present the numerical troubles typical of MIMO systems. Clearly the output in an arbitrary section is also due to the excitation on previous sections and the proposed HF-MIRA takes this into account.

The output relative to the first section for the modeled system depends only on the corresponding input force because no previous inputs were acting on the structure (causality relationship). For real systems, and even for very highly damped ones like a railway track, some output will be inevitably present before the passage of the force on the first section. This problem can be overcome by neglecting, once completed the reconstruction process, the early sections that have been used just to start the algorithm. The number of neglected sections depends on the global damping properties of the system, which is very important for the definition of the number of sections.

When multiple outputs are available, several estimations of the input force can be made, computing statistical properties of the solutions set; if the dispersion is low, it is then reasonable to suppose that the power spectrum of the estimated input is similar to the one of the real input because they give the same output and the reconstruction procedure is not ill-conditioned. The reduction of a system to a SISO one limits ill-conditioning problems.

In this paper, the application of the HF-MIRA to a system with one output is presented, without performing statistical calculations. Even in this case the algorithm behaves well, estimating correctly supplied inputs of several kind, as shown in Section 4.2.

4.1. INPUT SIGNAL RECONSTRUCTION ALGORITHM

The proposed HF-MIRA (Fig. 9) works under the hypothesis that the output signal due to unmeasured inputs and to noise is much less than the output due to the input forces ($A(q)y(t) \gg C(q)e(t)$). It does not use convergence iterations; the following steps are repeated for the n sections, starting from the first one ($sect = 1$):

(1) input estimation of actual section u_{rec} using only the corresponding local output y^* (equivalent to a SISO system) with the reconstruction process detailed later;
(2) estimation of the global output y_{sim} on the whole reconstruction period due only to this input;
(3) global output y^* updating subtracting from it the output u_{sim} estimated in step (2);
(4) move to next section ($sect = sect + 1$) and iteration until the last section has been reached ($sect = n$). The force signal is then reconstructed by linking together all inputs reconstructed in step (1).

The heart of HF-MIRA is the reconstruction process that manipulates zeros such that instabilities are prevented.

To this purpose equation (1) can be rewritten in a more convenient form as

$$A(q)y(t) = B(q)u(t) = B^{int}(q)B^{ext}(q)u(t) \tag{3}$$

where $B(q)$ has been factored in the polynomials $B^{int}(q)$ and $B^{ext}(q)$ that deal respectively with zeros internal and external to unitary circle in the z-plane (Fig. 6). Defining a new dummy time history $x_0(t)$ as

$$x_0(t) = B^{ext}(q)u(t) \tag{4}$$

it is possible to rewrite equation (3) as

$$A(q)y(t) = B^{int}(q)x_0(t) \Rightarrow x_0(t) = \frac{A(q)}{B^{int}(q)}y(t) \tag{5}$$

that describes the fictitious system shown in Fig. 10.

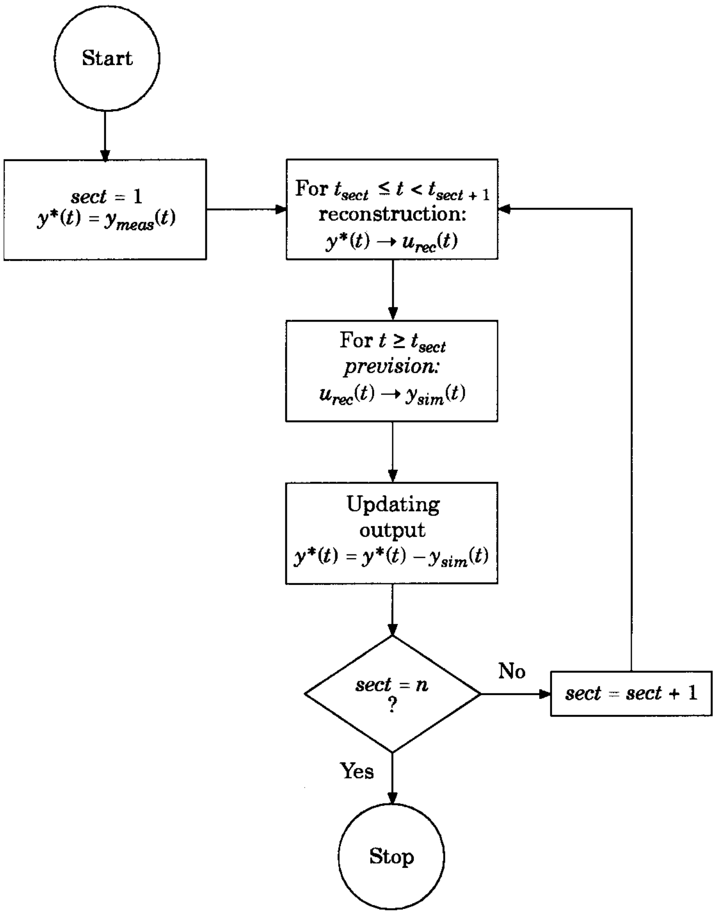

Figure 9. Block diagram of reconstruction algorithm (HF-MIRA).

The time history $x_0(t)$ can be calculated by using equation (5) without numerical problems as the poles of $A(q)/B^{int}(q)$ are stable by definition. Since $y(t)$ is real, and poles and zeros are real or complex conjugate pairs, $x_0(t)$ is real also. The direct use of equation (4) for the final determination of $u(t)$ is not possible as the transfer function $1/B^{ext}(q)$ is intrinsically unstable, but it can be rewritten in the form

$$x_0(t) = u(t) \prod_{i=1}^{nr+2*ni} (q - z_i^{ext}) \tag{6}$$

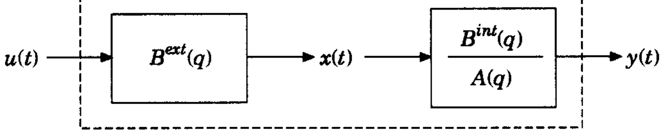

Figure 10. Block diagram of the system with $B(q)$ decomposition and auxiliary time series $x(t)$.

where the external zeros z_i^{ext} (relative to the considered section) can be grouped in nr real zeros and $2{*}ni$ complex conjugate zeros. The HF-MIRA treats the two cases differently, iteratively applying for the nr real zeros

$$x_{j-1}(t) = (q - z_j^{ext})x_j(t) = x_j(t+1) - z_j^{ext}x_j(t) \;\Rightarrow\; x_j(t) = \frac{x_j(t+1) - x_{j-1}(t)}{z_j^{ext}} \quad \text{with } j = 1, \ldots, nr \tag{7}$$

and for the ni complex conjugate zeros $z_j^{ext} = a_j + ib_j$

$$x_{j-1}(t) = (q - z_j^{ext})(q - \bar{z}_j^{ext})x_j(t) = x_j(t+2) - 2a_j x_j(t+1) + (a_j^2 + b_j^2)x_j(t)$$

$$\Rightarrow\; x_j(t) = \frac{x_{j-1}(t) + 2a_j x_j(t+1) - x_j(t+2)}{(a_j^2 + b_j^2)} \quad \text{with } j = nr+1, \ldots, nr+ni. \tag{8}$$

At the end of the procedure the force has been reconstructed, namely $u(t) = x_{nr+ni}(t)$.

The calculation of any $x_j(t)$ is made by using equations (7) and (8) back from the last sample t_b of the interval $[t_a, t_b]$. To determine the value at time t it is necessary to know the values at times $t+1$ (for real and complex zeros) and $t+2$ (only for complex zeros). To start the computation it is then necessary to give some 'initial conditions'; the numerically more obvious hypothesis is to suppose null these values, coherently with the fact that after t_b the force has left the actual section.

The choice of these initial conditions is then somewhat arbitrary and leads to a set of different solutions that nonetheless become identical after a very limited number of time intervals Δt, which is function of the number $nr + 2{*}ni$ of external zeros (i.e. the number of conditions to be assigned).

To minimise this problem, it is possible to start the reconstruction beyond the considered interval ($t > t_b$) even if this is not completely correct. Auxiliary samples that obviously belong to next section are in fact reconstructed by using the transfer function of the actual section. It must be considered anyway that the poles remain the same and only the zeros change moving from a section to the next one. Even if extensive analyses have not been performed, a clear improvement in the reconstructed force has been observed using only a few samples (slightly $> nr + 2{*}ni$, in our case $\cong 30$). For an easier implementation, HF-MIRA has been used with a number of auxiliary samples equal to the number of samples corresponding to the crossing of one section (128 samples).

4.2. EXAMPLES OF FORCE RECONSTRUCTIONS WITH HF-MIRA

The developed algorithm has been applied to the simulated outputs (Section 3.2) due to broad- and narrowband forces (Section 3.1). Figure 11 shows the 1/12 and 1/3 octave band autopower spectra obtained by synthesising constant percentage bands from the narrowband spectra of reconstructed time force signals and also shows the spectra of simulated inputs.

For the broadband force, the comparison of reconstructed and simulated input clearly shows how the HF-MIRA allows the reconstruction of the spectrum with a good fidelity. The frequency range around 1 kHz shows a large error in the 1/12 octave band diagram; this is mainly due to the absence of input signal in that range; the 1/3 octave band spectrum clearly shows a minor error, even if its use is critical below 400 Hz for the well-known synthesis problems.

The spectra for narrowband force are good around 1 kHz (where the frequency content is maximum), where the previous simulation proved to be poor. This proves that the model of the structure and the reconstruction algorithm work fine, provided there is some signal

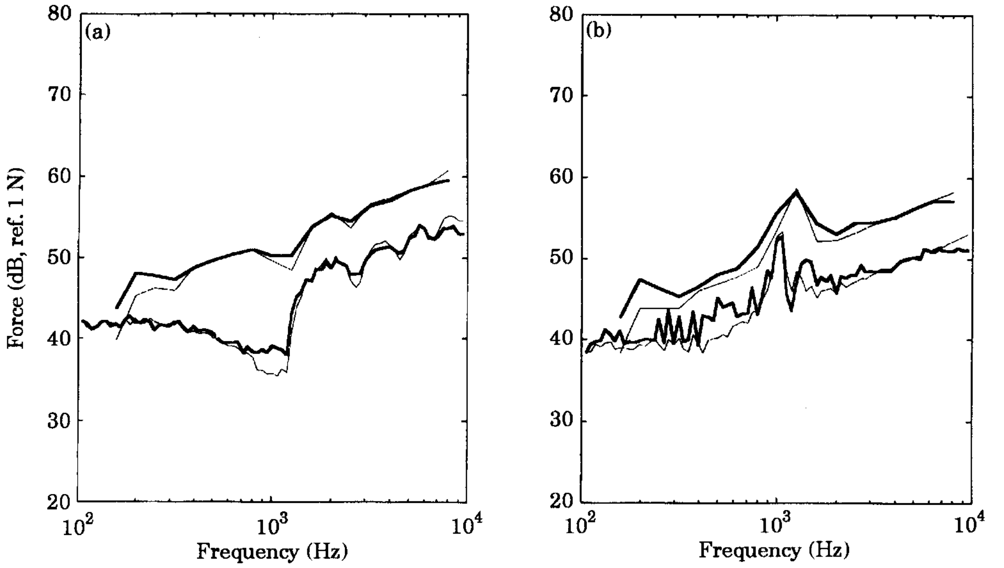

Figure 11. One-third and 1/12 octave band autopower spectra of input (——) and reconstructed (——) force. (a) Wideband force; (b) narrowband force.

to be reconstructed! The noise inevitably present in the measurement greatly overcomes this problem, and this is why random noise has been added to the 1-kHz pure sine signal. Obviously 1/12 octave band representation is more sensitive to low-amplitude background noise, and the diagram is not very regular; as usual the problem disappears with the 1/3 octave band rougher frequency resolution.

5. CONCLUSIONS

In this work, an algorithm for the reconstruction of a mobile input signal acting on a damped linear mechanical system has been proposed and successfully validated. Its application is advantageous, and sometimes results to be the only possible, for those systems in which usual reconstruction techniques are inapplicable or strongly ill-conditioned.

Proposed HF-MIRA is stable (no matrix inversion), does not diverge (solution found not iteratively) and is particularly fast (one-step). It has been applied to a complex mechanical structure identified through an ARMAX model, whose parameter extraction phase is not particularly heavy from a computation point of view. The critical step is the choice of the model that better represents the structure (ARX, ARMAX, ARARMAX, etc.); it must be performed by a skilled analyst using non-standard procedures that need many parameter extraction and validation tests.

The procedure has been validated by numerical comparison of supplied and reconstructed input signals. It proved to be stable and therefore can be applied to both narrow- and broadband mobile forces acting on complex structures. The work will be continued by applying HF-MIRA to systems with one mobile input and multiple output, using redundant output information to improve input reconstruction and to verify the input reconstruction repeatability. It will also be extended to systems with a limited number of mobile inputs and multiple output. In this case, the reconstruction of the input in a given time interval will have a number of unknowns equal to the number of mobile forces.

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