Vehicle System Dynamics, 30 (1998), pp. 1–15 0042-3114/98/3001-001$12.00

Time Domain Model of the Vertical Dynamics of a Railway Track up to 5 kHz

A. BRACCIALI, G. CASCINI and R. CIUFFI

SUMMARY

The modelling of the vertical dynamics of a track at high frequencies requires rather complex approaches to take into account section deformations. Validation is usually made by comparing computed frequency responses with measured ones. In this study an experimental model of a railway track is proposed based on the analysis of recorded time histories of impact excitations and the corresponding vibrations of the rail with autoregressive (AR) techniques. Measurements are used not only as a convergence parameter that the model must approach, but are also entirely used to describe the dynamic behaviour of the rail in the frequency range 150 ÷ 5000 Hz. Frequency response functions are reconstructed with a very high fidelity but the model obtained is not general, as it is applicable only to the measured track section under the hypothesis of linearity. The measurement details, the construction and the validation of the model are shown in this paper.

1. INTRODUCTION

The dynamics of the track has been acknowledged since the last century as being a fundamental topic. Since then, the simulation of the behaviour of the track at the passage of the train has been considered very important to estimate the stresses in the various elements that the track is made of and to limit the consequences of possible damages.

Nowadays mathematical models of the track are used for many different purposes: to study the dynamic behaviour of the vehicles, to investigate the effects of fatigue on the rails, to estimate rail and wheel wear, to simulate the behaviour of the single components (rails, railpads, sleepers, ballast, etc.), to estimate the propagation of the vibrations in the ground, to reconstruct input forces for noise emission prediction and so on.

Any mathematical model of a mechanical system is, from an engineering point of view, a relationship between the quantities (variables) that describe the system

[1] Dipartimento di Meccanica e Tecnologie Industriali, Università degli Studi di Firenze, via Santa Marta 3, 50139 Firenze, Italy.
Corresponding Author: Dr. Andrea Bracciali, Dipartimento di Meccanica e Tecnologie Industriali, Università degli Studi di Firenze, via Santa Marta 3, 50139 Firenze, Italy. Tel. +39 55 4796292 / 4796514. Fax +39 55 4796400. e-mail:bracciali@ing.umifi.it

in terms of algebraic or differential (system of) equations. A model can be built following two different paths. The first one describes the system as a collection of subsystems whose behaviour is known *a priori*; the laws that identify and describe such subsystems, once properly related between them, form the model of the complete system. This approach clearly does not require any observation of the real behaviour of the system: the data from experimental tests, if available, can be used to validate the model or to update unknown parameters with a manual or automatic convergence process. The other approach is known as the *System Identification* approach, and it consists in the construction of the model starting from the input and output signals measured directly on the system to be modelled.

The usual approach for railway tracks is the first one. It allows the separate study (and the subsequent assembly) of the model of the various components of the track. Its advantage is clear: any component can be separately optimized with *ad hoc* tests, and modularity and upgradeability of the model are always possible even using the results of different researchers. Found parameters are directly related to the single components, and critical components can be easily found with parametric simulations.

Analysed frequency range obviously depends on the use of the model. As trackside and on-board noise measurements highlight the good energy content of noise up to the 5 kHz octave band, models to be used in this field need a good fidelity of description of the track in this range. Unfortunately some problems arise at the upper limit that are negligible at lower frequencies. The infinite nature of the track, the non homogeneity of the supports, the presence of components with strong non-linear behaviour are only examples of the obstacles to be faced during modelling of the track.

An exhaustive review of railway track and vehicle-track interaction modelling can be found in [1]. From this paper it is easy to see how the more detailed description of the rail dynamics at high frequency is possible only by using sophisticated and complicated methods, as a simple beam model of the rail is unsatisfactory even by using a Timoshenko beam. For frequencies above 1 kHz, severe deformations of the cross section of the rail impair the use of such simple models. This leads to more complicated models that use a greater number of parameters that can no longer be theoretically identified anymore. These parameters can be found only with careful and costly comparison with experimental measurements and, more important, lose their "physicity". To give an example, stiffness and damping values for railpads found by several researchers differ by one or two orders of magnitude [1 - App. A].

As the loss of physical meaning intrinsic in the use of a great number of parameters appears inevitable, the System Identification approach has been followed in this research. A model has been built by using exclusively experimental data. Techniques used give not only an estimation of the measured frequency responses of the system, but also lead to an autoregressive parametric model able to represent correctly the behaviour of the track up to 5 kHz, viz. up to the typical noise emission frequency range.

2. EXPERIMENTAL STUDIES

Experimental data collection requires a special care for the purposes of the present work, as the measurements are used not only to *check* the model but to *build* it, with a procedure similar to the one used in experimental modal analysis. The fact that the amplitude range of the measurements is limited ($20 \div 30$ dB vs. > 60 dB for lightly damped structures) required special attention as responses to impulse forces are very short.

The track model presented in this paper will be used in an algorithm for wheel-rail force reconstruction, and the choice of input and output points is strictly related to this use of the model; the procedure is however not affected at all by this selection and maintains its validity. The study is limited to the vertical response under vertical excitations in the frequency range $200 \div 5000$ Hz; as the vibration transmissibility between the rails (through the sleepers) is very low in this frequency range [7 - App. G], only one rail 3900 mm long has been analysed. The description of a double length of the rail (7800 mm) is immediate if symmetry conditions are retained as valid.

Measurements have been made on a conventional ballasted track of the Florence-Rome line, about 2 km far from main Florence Santa Maria Novella railway station. As the line was normally operated during tests, these have been made between train passages using an impact hammer instrumented with a piezoelectric load cell that can be easily used to give excitation in different sections. This kind of excitation is closer to the real one (due to the interaction of wheel roughness and/or wheelflats with the roughness of the rail) than an equivalent given by a shaker. Crest factors (and hence nonlinearities) do not constitute a problem as measurements made with forces with ratio 1:100 give, apart for the different contribution of ground noise, the same frequency responses. Hammer tip was sufficiently rigid to give energy (almost flat autospectrum) up to 5 kHz (Fig. 1). Measurements have been made with force peaks of about 5 kN; this dynamic value, clearly lower than the static load for typical european coaches and locomotives ($100 \div 200$ kN/axle), agrees with values estimated by Ten Volde and Van Ruiten [6] for vertical dynamic wheel-rail forces for speeds between 80 and 120 km/h. The characterization of the track with force of the same order of magnitude of real ones greatly prevents non linear effects. For the forementioned "boundary conditions" it has not been possible to take measurements with a static load (weight) applied to the track, a limitation that can lead to not completely correct results, but that can be easily removed in subsequent measurement campaigns.

For the model to be reliable in the whole frequency range, it is necessary that the input/output signals used to build it are as general as possible, i.e., they must provide the most complete information on the dynamic behaviour of the system. As impacts are particularly suitable for this goal (very broad band excitation), possible lack of output signals can be attributed to the behaviour of the system and not to an insufficient excitation.

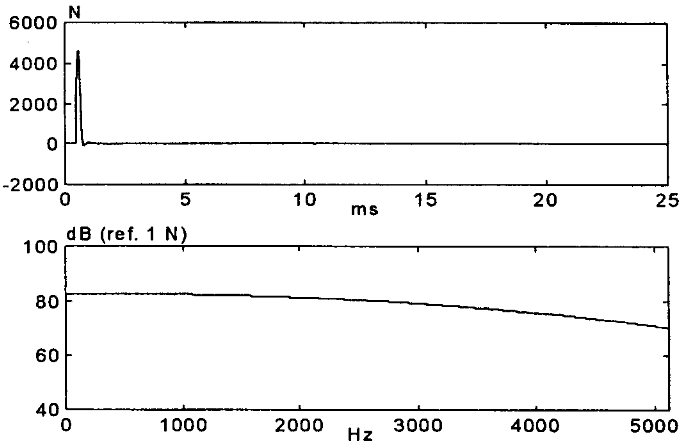

Fig. 1. Time history and spectrum magnitude of an impact excitation. Time axis is limited to 512 samples, while the complete acquisition lasts 2048 samples.

For the measurement of the response of the system, a section of a rail has been instrumented with small quartz accelerometers mainly for frequency range reasons. The instrumented section is 120 mm away from the centreline of a sleeper, because during previous measurements the application of sensors just over the sleeper or in the midspan between two sleepers proved to provide insufficient information during train passage. This becomes evident considering that the first vertical flexural frequency is at around 1 kHz (the so called ''pinned-pinned resonance'') and that it has nodes on the sleepers and maximum amplitude in the

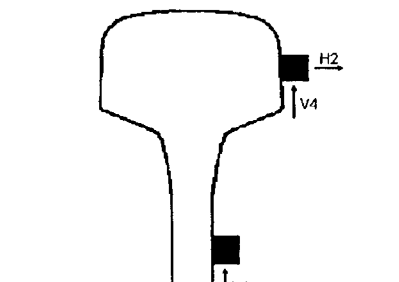

Fig. 2. Vertical (V) and horizontal (H) accelerometers location in the instrumented section. Only vertical accelerations V1-V4 are considered as the model outputs.

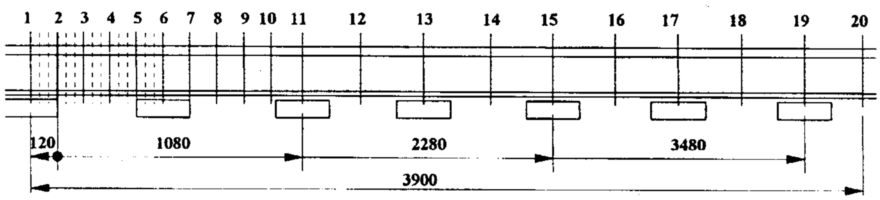

Fig. 3. Analysed track section. Measurements have been made by applying forces in all sections but, as variation is small for very close input sections, only numbered sections have been used for model construction. Accelerometers have been placed in section #2 (thick) according to Figure 2. Sections used for delay evaluation are #2, 11, 15 and 19, for which relative distances are indicated (see Fig. 4). Section #6 has been used for the comparison between measured and reconstructed FRFs.

midspan between two sleepers with the obvious consequences on the amplitude of measured accelerations. Six piezoelectric accelerometers, with a mass of 10 grams and integrated electronics (ICP®), have been glued with cyanoacrilate (mounted resonant frequency $\gg 5$ kHz) on the rail with mica washers interposed for electrical insulation from signalling and return currents (Fig. 2).

The excitation has been given in 30 points on the railhead (Fig. 3), closer in the span where measuring section is located, obtaining this way a very detailed description of the behaviour of the rail for the first two spans. For the other five spans a coarser description has been considered to be sufficient, at least for the more characteristical points of excitation (over a sleeper and in the midspan). As the track is naturally infinite and moreover highly damped, it acts as a guide for the elastic waves that partially leak through the supports. It follows from this that increasing the distance from the input point the response is more attenuated, up to 3 dB/m [5]. The force has been given vertically five times on the head of each section.

Signals have been collected, conditioned and recorded using a PC equipped with a National Instruments© general purpose I/O board. Gains have been individually selected for each channel, with the best use of the 72 dB amplitude range of the A/D converter. Sampling frequency was 20480 Hz, and all the settings have been made by using Virtual Instruments generated with the LabVIEW® software.

3. DYNAMIC MODELLING OF THE TRACK USING SYSTEM IDENTIFICATION

The description of a system using System Identification techniques is made with the following steps:

i. experimental data collection;
ii. choice of the type (structure) of the model;
iii. model parameter estimation;
iv. model validation.

The first step is described in the previous paragraph; the second step is critical as the choice of a non optimal structure for the representation of the system to be modelled leads to an inevitably mistaken identification process. Unfortunately general rules to be applied for the choice of the best structure do not exist, and the experience of the analyst is the only resource that is possible to use in this phase.

The different model structures have been considered suitable or not for this research depending on their capability to reproduce measured signals. In this work the comparison between the models has been made by using the software libraries contained in the Matlab© Identification Toolbox [4]. Model parameters are estimated with the *prediction error method* which iteratively reduces the least squares error made in the representation of the behaviour of the system. It is worth reminding that the third step leads *anyway* to a result: the necessity of the model validation step (iv) is therefore clear, and it must be conducted with extreme attention to categorically confirm the capability of the model to tackle the goals that it is requested to satisfy.

Several reasons can lead to the lack of validity of a model [2]:

i. the numerical procedure for the determination of the best model with the used criterion failed;
ii. the criterion used to compare the various models is not correct;
iii. the structure of the model is not appropriate, i.e. it is not capable to give a sufficiently good description of the system;
iv. data set used for the definition of the best model does not contain sufficient information.

The most important phase of model tuning consists in fact in solving these problems, with the so-called "System Identification Loop".

Not all the data collected in the experimental phase have been used for modelling the vertical rail dynamic behaviour: signals from accelerometers 1,3,5 and 6 have been considered sufficient. From the analysis of measured FRFs it emerged that the description of the span where measurement section is located is even too fine, and for the sake of simplicity the number of inputs has been limited to 20 (see Fig. 3 again).

At the end of the modelling phase, a set of poles and zeros are obtained identifying the time response of the system. While the poles relative to an output are the same for any input, there is no link between the parameters that define the transfer function of two different outputs: it is therefore possible, to reduce computation times and to optimize the response, to build different models for different outputs.

The matrix of the experimental data has been organized in 21 columns, the first containing the output vector for an accelerometer and the remaining the input vectors, that are made of the set of the excitations given on the corresponding sections, opportunely padded with zeros so that simultaneous excitations on several inputs are avoided. The complete set of the responses has then been rearranged to fill the output vector. Only four of the five available samples for each section have been used to build the model; the fifth I/O sample has been left for the validation of the model. This form of data representation is clearly memory expensive. A different way has been tried, putting the input data on the last 20 colums without any zero padding and filling the first column with the sum of all the outputs using the superposition principle. Unfortunately this very compact representation failed to give consistent results in the parameter estimation phase, most probably because the excitations on all the inputs are almost equal and then it is impossible to reasonably assign to each input the corresponding part of the output signal.

A sufficiently generic model structure capable to describe the dynamic behaviour of a mechanical system is [2]:

$$A(q)\,y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t), \tag{1}$$

where $y(t)$ is the system output signals vector, $u(t)$ is the system input signals vector, $e(t)$ is the uncontrolled inputs vector, measurable or not (noise), q is the *forward shift operator* for which holds

$$\mathrm{q}\; u(t) = u(t+1),$$

and A(q), B(q), C(q), D(q), F(q) are polynomials matrices in the *delay operator factor* (or *backward shift operator*) q^{-1}.

The hypotheses under which the procedure can be applied are that (i) the system is linear, (ii) time invariant, that (iii) the inputs are deterministic (or at least partly deterministic), and that (iv) uncontrolled inputs and all the disturbances can be described as random variables. From (iii) and (iv) it follows that the outputs are stochastic processes with deterministic components. Linearity hypothesis (i) is clearly an idealization of the behaviour of the system, but the advantages that it introduces amply justify its adoption; the other hypotheses are satisfied by the system.

For the choice of the model structure some considerations must be made. An hypothesis that is always verified for mechanical systems is that the poles of the transfer function relative to an output and associated to the various inputs are equal; in this case F(q) reduces to the identity matrix. Conversely C(q) and D(q) cannot be omitted as $e(t)$ is a white noise that is used to take into account the uncontrolled input contribution; rail acceleration measurements are contaminated by electric traction return currents, signalling currents and, more generally, by all electro-magnetic interferences. It is obvious that poles and zeros relative to impact excitations are completely independent of those related to disturbances. To confirm these assessments other simpler structures have been tried, as ARX (A(q), B(q)) or ARMAX (A(q), B(q), C(q)), but with less satisfactory results.

The portion of track considered in this research is modelled as a 20 inputs, 4 outputs MIMO system. As previously stated, the absence of links between the parameters that define the transfer function of two different outputs is such that system is equivalent to four 20 inputs, 1 output MISO system. For a general MISO system with *nu* inputs, Equation (1) is scalar and $A(\mathrm{q})$, $B(\mathrm{q})$, $C(\mathrm{q})$ and $D(\mathrm{q})$ are the following polynomials in q^{-1}:

$$A(\mathrm{q}) = 1 + a_1\mathrm{q}^{-1} + a_2\mathrm{q}^{-2} + \ldots + a_{na}\mathrm{q}^{-na}$$

$$B(\mathrm{q}) = b_1\mathrm{q}^{-nk} + b_2\mathrm{q}^{-nk-1} + \ldots + b_{nb}\mathrm{q}^{-nk-nb+1}$$

$$C(\mathrm{q}) = 1 + c_1\mathrm{q}^{-1} + c_2\mathrm{q}^{-2} + \ldots + c_{nc}\mathrm{q}^{-nc}$$

$$D(\mathrm{q}) = 1 + d_1\mathrm{q}^{-1} + d_2\mathrm{q}^{-2} + \ldots + d_{nd}\mathrm{q}^{-nd}$$

where a_i, c_i and d_i coefficients are scalar, while b_i coefficients are $1 \times nu$ vectors; at any input a time delay nk is assigned, related to the number of discrete time samples that pass between the beginning of the input and the beginning of the output.

The number of parameters (na, nb, nc, nd) must be defined before the construction of the model. As it is clear from (1), this is equal to define the number of zeros and poles for both measured and uncontrolled inputs. A guideline for the choice can be obtained from the observation of the experimental and the modeled transfer functions of the system; the analyst must then recognize if the model is too complex or too simple, and consequently decrease or increase the number of parameters.

It is worth mentioning that the definition of the delays associated to each I/O couple has presented some peculiarities. For a mechanical system the delay can be evaluated considering the physical meaning of this parameter, i.e., the number of sampling periods that last from the application of the force to the appearance of the output. For inputs given in the sections close to the transducers, the delay is reasonably related to the speed of propagation of elastic waves in steel. For distant sections this relationship does not hold any more, and the delay increases more than proportionally with the distance (Fig. 4). The analysis of this phenomenon lies beyond the scope of this study, even if it would be worth investigating; a first hypothesis (to be confirmed) could be that the vertical impact excitation induces elastic waves that are subjected to many reflections on the boundary of the rail, and the propagation path results to be longer. Delays assigned to each I/O couple of signals have been easily determined by visual inspection of time histories thanks to their particular shape; such an analysis would have been harder for a shaker excitation as the propagation properties of the system are not equally clear in this case.

The stability of a model is strictly related to the *causality* of the response of the system, that must be zero in absence of excitation. While this hypothesis is generally physically satisfied, passing to discrete-time representations it is possible that input and output peaks are simultaneous if sampling time is too long or the sections are very close. This happens to the track when the force is given in the vicinity of the accelerometers. To prevent this pseudo-loss of causality, it proved

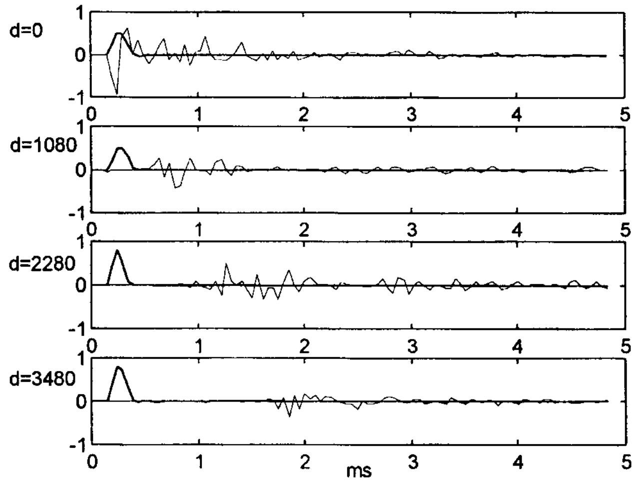

Fig. 4. I/O delays for excitation on sections #2, 11, 15 and 19. Acceleration (thin line) measured in V1 starts at a time that is more than proportional to the distance between the measurement section and the force (thick line) application section (d, mm). Signal amplitudes have been normalized for comparison.

to be sufficient to artificiously shift output signals so that there is at least one delay. This operation introduces only a change of phase without modifying the global properties of the system, and has been taken into account in the validation

Table 1. Number of parameters used for ARARMAX model construction.

OUTPUT	V1	V2	V3	V4
na	54	58	60	54
nb - input 01	52	56	58	52
nb - input 02	52	56	58	50
nb - input 03	52	56	58	50
nb - input 04	52	56	58	52
nb - input 05	50	56	58	50
nb - input 06	52	56	58	50
nb - input 07	50	56	58	50
nb - input 08	50	56	58	50
nb - input 09	50	56	58	50
nb - input 10	52	56	58	50
nb - input 11	52	56	58	52
nb - input 12	52	56	58	50
nb - input 13	52	56	58	52
nb - input 14	50	56	58	52
nb - input 15	50	56	58	50
nb - input 16	52	56	58	50
nb - input 17	52	56	58	50
nb - input 18	52	56	58	52
nb - input 19	52	56	58	52
nb - input 20	50	56	58	52
nc	5	5	5	5
nd	5	5	5	5

im

phase where measured and reconstructed frequency response functions are compared.

The model developed is valid to describe the behaviour of the rail up to 5 kHz. An extension to higher frequencies is possible if experimental data are still valid: it should be considered anyway that the number of parameters of the model could increase so much that computation time becomes unaffordable. This problem could be prevented by using a simple trick, i.e., by filtering I/O sequences to share the frequency range and separately extract the parameters for each sub-range. Poles and zeros could then be combined to obtain the parameters of the complete system. For this study this operation did not prove to be necessary; a signal filtering and a decimation have been done anyway to limit the length of signals to be processed and then to reduce computation times. These filtering operations have been performed without phase distortions, that are absolutely incompatible with the following manipulations of time histories.

The final model is made of 4548 parameters for the representation of the transfer of the measured inputs to the outputs and of 200 supplementary parameters for the description of uncontrolled inputs (Table 1). These figures are very reduced if compared to the enormous number of information that the model is capable of representing: an analogous model that uses the 80 FRFs needed to completely describe the system (20 inputs by 4 outputs), with the typical 800 lines resolution, requires 64000 complex numbers, i.e., 128000 parameters.

4. MODEL VALIDATION

The model of the track has been submitted to a careful validation process made of different steps.

Step 1: verification of the capability to simulate the dynamic behaviour of the system under an excitation not used for the construction of the model (Fig. 5).

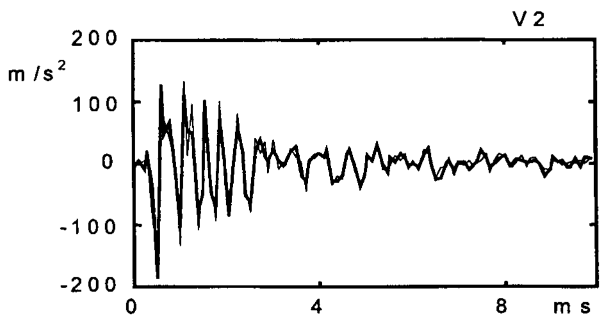

Fig. 5. Model simulation of the rail response under a vertical impact excitation, not used for the model parameter estimation, applied on the measurement section. Comparison between the accelerations measured in V2 (thick line) and the response predicted by the ARARMAX model (thin line).

This verification can be considered sufficient, even for this test to be really reliable a completely different excitation (for example with a shaker) should be used.

Step 2: comparison of measured and estimated FRFs (Fig. 6). While the phase reconstruction is almost perfect, some small modulations in the amplitude are lost in the reconstruction process, a problem that can however be prevented increasing the number of parameters in the model. This non perfect reconstruction has also some advantages, as it does not present the typical irregularities of measured FRFs: while *all* the parameters of the model are responsible for the definition of the FRF, frequency lines are completely independent in the measurements. Where the signal/noise ratio is particularly low, the estimation of the transfer function becomes wrong with traditional techniques based on *local* parameters extraction; the obtained model is instead *global* and it is influenced by the behaviour of the structure in *all* the frequency range, also in those lines that are neglected by the other methods. Global methods are particularly efficient in highly damped systems, where, for example, it has no sense to talk about modes and the behaviour is not governed by a linear combination of eigenmodes as for lightly damped structures.

Step 3: control of the stability of the model, i.e., the prevention of possible divergence problems. It can be easily proven that the model is stable if the poles of

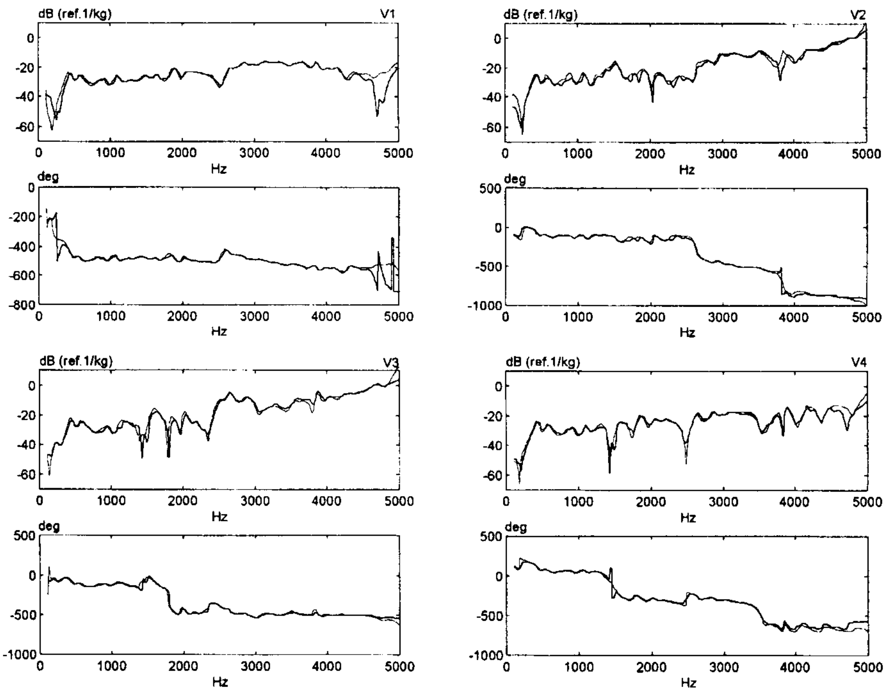

Fig. 6. Comparison of some measured (thick line) and reconstructed (thin line) FRFs for the four outputs used. Input is given on the instrumented section (#2).

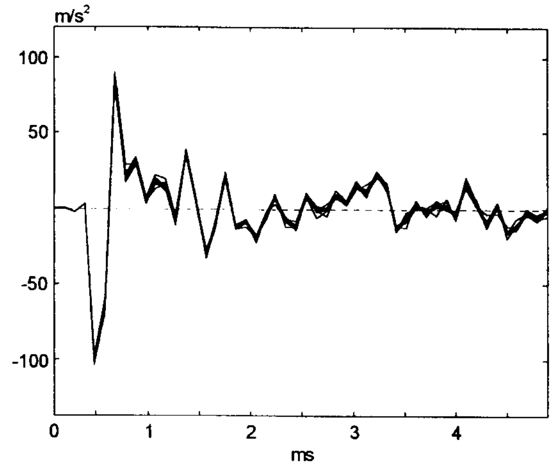

Fig. 7. Simulation of the rail response V1 under a vertical impact excitation on the measurement section #2 made by different models obtained randomly varying the ARARMAX parameters inside a $\pm\sigma$ confidence interval.

the transfer function, i.e. the roots of the polynomial $A(q)$, lie inside the unitary circle on the z-plane. The condition is verified for all the poles of the model of the track; the poles relative to the V1 output are shown in Figure 9.

Step 4: control that the order of the model (viz. the number of parameters) is not too high. If in the vicinity of any pole of the transfer function one or more zeros are present, a mutual cancellation occurs and number of parameters of the model can be reduced without significantly affecting the FRF, at most smoothing an "ondulation". In the same way it is allowed to substitute poles or zeros particularly close with only one parameter.

Step 5: calculation of the indetermination of the model, i.e. the evaluation of the statistical uncertainty (standard deviation) of the estimated parameters on the response. A common estimator of the uncertainty is the least squares error between the response estimated with "nominal" parameters and the one estimated with "perturbated" parameters. For this goal, several substantially equivalent verifications have been made:

i. direct numerical analysis of the relative uncertainty: this is a faster and easier test, but it is hard to really understand the effect of "uncertain" parameters;
ii. sensibility analysis of the simulated response under any real or fictitious excitation: the track behaviour is simulated by randomly varying the parameters of the model inside a confidence interval of $\pm\sigma$. Fig. 7 shows an example of this analysis, that shows how the model has a very low indetermination;
iii. sensibility analysis of the transfer function of the model at the variation of the parameters in the range defined by $\pm\sigma$: this analysis proves to be particularly efficient beacuse it is easy to appreciate the effect of the uncertainty of the parameters on estimated transfer functions (Fig. 8);

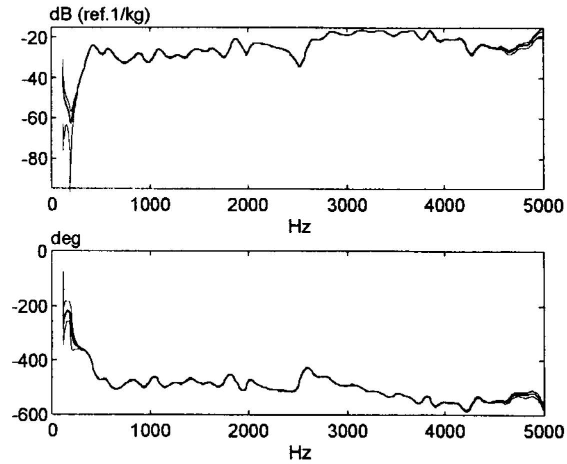

Fig. 8. Magnitude and phase of model point FRF H_{VI} (thick line) and uncertainty with parameter variation in the range defined by $\pm\sigma$.

iv. sensibility analysis of poles and zeros of the model: a visual check can be obtained by plotting on the z-plane the zones corresponding to $\pm\sigma$ around each parameter (Fig. 9).

Step 6: use of the models in the form (1) to estimate the effect of uncontrolled inputs on the response of the system. Many simulations have been performed (Fig. 10) giving to the system an input made of a white noise superimposed on the other normal input forces. The variance of the disturbance signal is the one estimated by

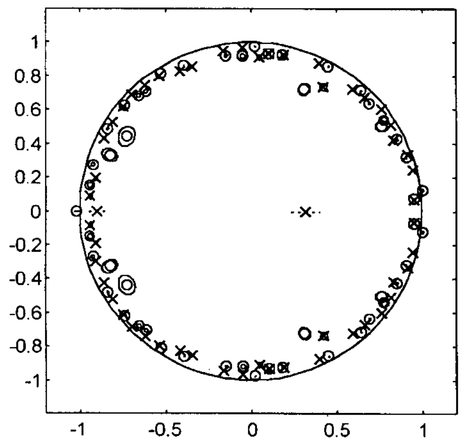

Fig. 9. Z-plane representation of poles ($\times$), zeros (o) of the model point FRF H_{VI}. The confidence regions corresponding to one standard deviation are indicated with thin line ellipses (that can be smaller than the symbol "o").

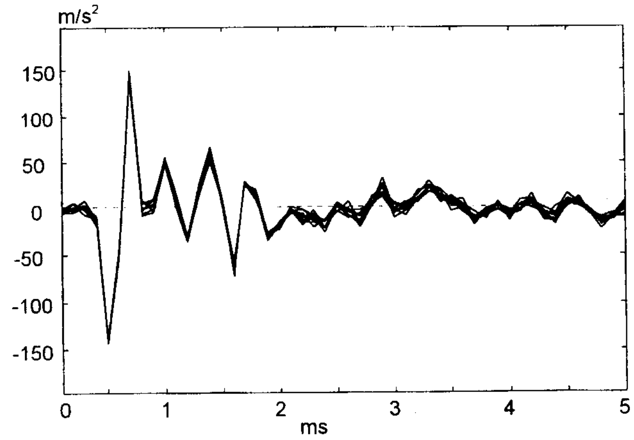

Fig. 10. Simulation of the rail response V1 under a vertical impact excitation on the measurement section #2 superimposed to a white noise acting as uncontrolled input. The model parameters vary randomly within a confidence interval of $\pm\,\sigma$.

parameter extraction algorithm. If the response is strongly conditioned by the presence of the disturbance, it is impossible to relate the system output exclusively to measured inputs and the model itself loses its validity as the experimental data used for its construction have probably been misinterpretated.

5. CONCLUSIONS AND FURTHER DEVELOPMENTS

This work has described the procedure to obtain a model of the vertical dynamic of a railway track up to 5 kHz. System Identification techniques applied to experimental data from an *ad hoc* measurement campaign results in a particularly efficient ARARMAX model, that brilliantly passed a careful validation phase.

The model will be used in an algorithm for the reconstruction of wheel-rail contact forces based on the HF-MIRA technique described in [8]. In order to obtain the best results in the contact forces estimation, a formally identical ARARMAX model could be constructed by using more suitable experimental data collected with track statically loaded, for example with a locomotive or a coach.

With absolutely identical procedures it is possible to extend the model to the horizontal dynamic of the track, to longer portions of the track or to higher frequencies, taking into account that the validity of experimental data must be absolute and that computation times can increase greatly. While CPU resources problems can be overcome with the procedure outlined above, there is no possibility to correct in the analysis phase data that have not been properly acquired.

Compared to other approaches found in literature, the one adopted here does not introduce any schematization of the behaviour of the system: the *global* use of experimental data guarantees the total accordance with reality. Moreover the ARARMAX model has not the limits of a simple FRF of the system, but can be used in a wide variety of applications, for example the simulation of the behaviour of the system subjected to an arbitary number of inputs, that is performed very quickly as time signals are very simply processed (the response is a sum of products) directly in the time domain, without the well known problems (windows, leakage,...) of Fourier transforms. Furthermore both the estimation of the uncertainty of the simulation and the evaluation of the effect of disturbances on the output are immediate.

The reconstructed FRFs follow with very good fidelity the measured ones both in amplitude and phase, and are even more robust at those frequencies where the system response and coherence are low.

Modelling is not influenced by the degree of damping of the system; once extracted, poles and zeros can be easily converted into the more familiar representation in terms of resonant frequencies and damping.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the Italian Railways FS for the authorization to perform described measurements and MURST and CNR for their financial support.

REFERENCES

1. Knothe, K.L. and Grassie, S.L.: Modelling of railway track and vehicle/track interaction at high frequencies. Vehicle System Dynamics 22 (1993), pp. 209–262.
2. Ljung, L.: System Identification: Theory and Practice. Research Studies Press Letchworth: England, 1987.
3. Pandit, M. and Wu, S.M.: Time Series and System Analysis with Applications. John Wiley and Sons: New York, 1983.
4. Matlab System Identification Toolbox Guide. The Mathworks Inc.: Natick, MA, 1992.
5. Remington, P.J.: Wheel/Rail Noise — Part I: Characterization of the Wheel/Rail Dynamic System. Journal of Sound and Vibration 46 (1976), pp. 359–379.
6. Ten Wolde, T. and Von Ruiten, C.J.M.: Sources and Mechanisms of Wheel/Rail Noise: State of the Art and Recent Research. Journal of Sound and Vibration 87 (1983), pp. 147–160.
7. Thompson, D.J.: Wheel-rail noise: theoretical modelling of the generation of vibration. Ph. D. Thesis, University of Southampton, 1990.
8. Bracciali, A. and Cascini, G.: High frequency mobile input reconstruction algorithm (HF-MIRA) applied to forces acting on a damped linear mechanical system. Mechanical Systems and Signal Processing v.12, n.2, March, 1998, pp. 255–268