

# AN ALGORITHM FOR ENERGY ROLLING CONTACT FORCE RECONSTRUCTION

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## ABSTRACT:

The knowledge of forces at the wheel-rail contact point is fundamental to estimate the consequences that it has in terms of noise and vibration. The traditional use of strain gauge mounted on the wheel web and axle is not capable to determine the high frequency content of contact forces. Inversely the measurements made on the rail are characterized by the spatial variability of input-output transfer functions. In this research the problem of rolling contact force reconstruction has been approached and solved through the following steps: (i) the track has been finely characterized for a finite length with the time series analysis of impacts supplied with an instrumented hammer by using an ARMAX model that proved to be capable to model the vertical rail dynamics up to 5 kHz; (ii) the response of the rail has been simulated with a random force acting on the system, and the variability of the transfer function has been properly taken into account by distributing the force on adjacent elements; (iii) the simulated response has been compared with the rail acceleration measured at the passage of several trains; (iv) the wheel-rail contact force has been estimated with a closed-loop algorithm.

It has been possible to reconstruct 1/3 octave frequency bands of contact forces with a simple and stable iterative procedure. Forces reconstructed from different sensors resulted practically the same for a given wheel, forces from nominally equal wheels are statistically compared and the first results of comparison of different rolling stock are shown.

## KEYWORDS:

rolling force, force reconstruction, wheel-rail contact force, ARMAX model

## 1. INTRODUCTION

In some cases the direct measurement of the dynamical forces that act on a system can not be performed with usual means and transducers: typical examples are the forces distributed over the system, the inaccessibility of the point of application and the mobility of the force. In these cases it is even impossible to use a force transducer and the estimation of the magnitude of the force can only be made through indirect measurement techniques.

A typical problem, particularly complex and not yet completely solved, is the estimation of the wheel-rail contact force. Rolling forces mainly determine vibrations in both the wheels and the rails that radiate noise in the environment. These noise sources are the main (over pantograph, running gear and aerodynamic noise) in the speed range 50÷250 km/h, and their frequency distribution shows high components up to around 5 kHz. For this reason it is particularly interesting to determine rolling contact forces at high frequency, and both the measurement chain and the reconstruction algorithm must be absolutely reliable.

Contact force reconstruction can be performed by using output (acceleration) signals measured on the rolling wheel or on the rail. In both cases there are some difficulties: for the accelerometers mounted on the wheel the signal output can be considered quasi-stationary, at least for small time intervals, but the instrumentation is particularly expensive as a telemetry system with a quite high transmission frequency is required; for the rail the phenomenon is typically transient, and this extremely complicates the experimental data processing to reconstruct the force. Despite this, the second way has been preferred not only for economical reasons (as it only requires "static" instrumentation) but as its application is more useful in perspective, as it allows the immediate comparison of contact forces due to different wheels of the same train and due to different rolling stock. Moreover, it represents a simple and cost effective approach to monitoring of wheel tread conditions.

Authors developed a completely general algorithm for the high-frequency reconstruction of a mobile excitation by using the system properties and the output under a moving input [1]; in this work a faster, simplified and particularly stable procedure for the reconstruction of the contact force 1/3 octave band spectrum is presented.

## 2. SYSTEM IDENTIFICATION

Railway tracks have some peculiarities that make their modeling almost impossible with the techniques traditionally used in mechanical systems analysis:

1. rails are "infinitely" long such that they behave, relatively to vibration propagation, as waveguides;
2. rail cross section undergoes deformations in its plane at frequencies above around 1 kHz, such that its modeling as Euler-Bernoulli or Timoshenko beams is not effective;
3. sleepers act as discrete supports and at the so-called *pinned-pinned resonance* they coincides with the nodes of the longitudinal deformation;
4. the global damping of the system is very high, with attenuations of 1÷5 dB/m along the rail as the rail pads dissipate energy and the ground below the sleepers behaves like a damper;
5. some elements, like the ballast and the rail pads, are only roughly considerable as a lumped spring-damper pair.

Mechanical systems modeling is usually made by using lumped parameters models (mass-spring-damper) or with non-parametric models through the input-output relationships in the frequency domain (Frequency Response Function - FRF) or in the time domain (Impulse Response Function - IRF).

The first approach, known as the modal approach, has the great advantage that supplies values that are directly related to the physical properties of the system, but it's not applicable to any kind of structure. If the system has highly coupled modes, high damping or numerous local modes (typically at the higher frequencies) it is impossible to extract modal parameters. This method is not applicable at all to railway rails, as they don't have modes (they are infinite in nature, and eigenmodes arise only for boundary

reflections), but simply waves!

The second approach is instead absolutely general and proves to be particularly efficient to model the output response in any measured degree of freedom for any input (real or simulated) applied to the system. Nonetheless the input reconstruction from output data in the frequency domain is almost impossible for ill-conditioning problems during the FRFs matrix inversion; moreover this approach shows other limits in the time domain (a windowing is necessary to reduce leakage errors) when the time series have variable lengths (the use of DFT is required instead of FFT).

For the representation of the dynamic behaviour of the track an ARMAX model has been set up (Fig. 1). The main properties of the model are summarized here (for a detailed description of the model and of its set up see [2]):

- the model is parametric, i.e. it describes the behaviour of the system with a finite number of parameters that are not related to the physical properties (mass, damping, stiffness) of the system but that are only a measure of how input signals are “transformed” into output signals;
- the parameters are extracted by using overdetermined information, so ensuring the highest reliability of the estimation; it is moreover possible to measure the parameters indetermination and, with appropriate model validation techniques, to reduce its value;
- it is possible to estimate the effect on the output of not measured inputs (noise); if this effect is small it means that the output only depends on the input and that it is possible to estimate the inputs from the (known) output, otherwise a non negligible contribution would be ascribed to the unknown input when inverting the noise inevitably present in the output;
- the number of the parameters that describe the system is small; the model is more stable with respect to FRF-based models where each spectral line is estimated independently from the others (high local errors can result even in models derived from very accurate measurements and processing), performing a kind of “smooth” of FRFs, thereby neglecting local errors.

Experimental track data were collected during a specifically conducted test campaign held in January 1996 on the Firenze-Roma line, close to the main Firenze Santa Maria Novella station. The rails are UIC 60, mounted on concrete sleepers laying on a ballasted bed. The model is able to represent with high fidelity the dynamic behaviour of the line segment up to 5 kHz. Accelerometers mounted on the section for track characterization were also used to collect measurements at the passage of around 70 passenger trains of different composition.

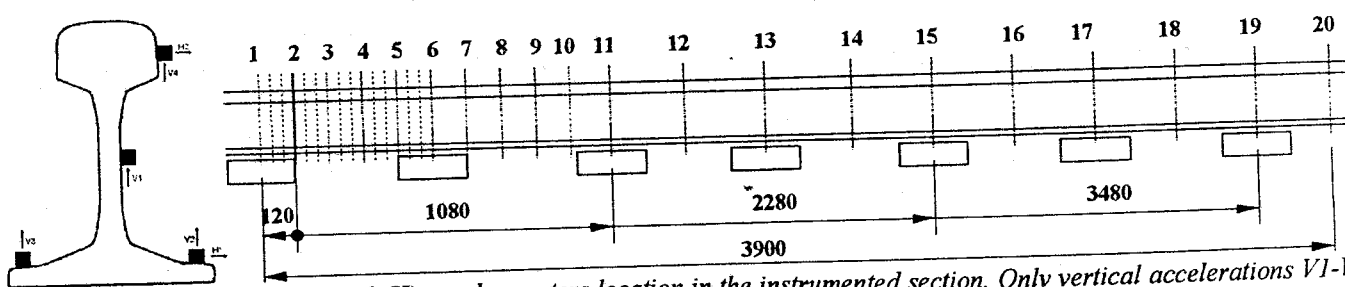


Fig. 1. Vertical (V) and horizontal (H) accelerometers location in the instrumented section. Only vertical accelerations V1-V4 are considered as the model outputs (left). Analysed track section. Measurements have been made by applying forces in all sections but, as variation is small for very close input sections, only numbered sections have been used for model construction. Accelerometers have been placed in section #2 (thick).

### 3. CONTACT FORCE RECONSTRUCTION

Many authors developed analytical and experimental procedures to estimate rolling contact forces from the measurement of the output, i.e. the acceleration measured on the rail. For example, in 1983 Ten Wolde e Van Ruiten [3] published their results obtained by using the point rail FRF and considering a time interval centered on the moment when the rail passes over the instrumented section. The contact

force frequency spectrum was estimated by the product of the inverted point FRF and of the output frequency spectrum. It is clear that this technique, used also by one of the authors in a previous work [4], does not take into account at all the contact forces before and after the passage over the instrumented section except for those in a close span where, moreover, the FRF considerably changes. Data in [3] are particularly useful, for the analyzed frequency range and for the considered rolling stock, as they are a basis for the evaluation of the quality of the results found with the new approach described here.

In this research the mobility of the excitation and the spatial variability of the transfer function of the system are considered. Due to the very high damping of the system, only one wheel is considered at a time. This assumption is justified when correctly choosing the wheel whose force is going to be reconstructed, i.e. by selecting only the internal wheelsets (wheelsets #2 and #3 on usual 2-bogies coaches), with a minimum overestimation of the force due to the contribution of adjacent non considered wheels on rail acceleration.

Reconstruction algorithm uses the ARMAX model of the track to simulate the response of the system to a known force that moves with the same law of motion of the considered wheel. By comparing the simulated output to the measured response under the real input a correction to the input force that reduces their difference is applied. The hypothesis is that the force that produces a response equal to the real response is equivalent to the real force. It is important to underline that in this approach we focused our attention only on energy content of forces and accelerations: comparison are made between 1/3 octave band frequency spectra. This representation of the signals doesn't provide any phase information, but this doesn't represent a limit for the application of the procedure, as signals considered here are basically random in nature.

The whole procedure needs the response measured in just one point on the rail. As the measurements were performed in 4 different points (all of them were used to build up the ARMAX model of the track), it is possible to independently reconstruct four times the unknown force from the signals of the different accelerometers. Comparing these four estimates for the same wheel it is possible to evaluate the reliability of the estimation itself.

### 3.1 The algorithm

The algorithm consists of the following steps:

1. a *pseudo-random* signal with flat frequency spectrum is generated several times with variable random phase. The amplitude of the signal is given by the ratio between the average level of the measured acceleration and the average level of the transfer functions of the system. These signals roughly represent the wheel-rail contact force. As the real phases of the input system are unknown, the output estimation is repeated with signals of the same amplitude but with different random phases and averaging the magnitudes to extract energy information. The output energy estimation is stable averaging 30 different-phase input forces;
2. with the technique described in the next paragraph the response of the rail at the passage of the wheel is estimated, supposing that the contact force has the trend defined at step 1;
3. from the set of the 30 simulated responses the 1/3 octave band output of the system is evaluated. For this computation only the portion of the signal related to passing sections 14÷1 was used, as the output in this interval is reasonably only due to forces acting on the modeled track span (it contains energy also from sections 15÷20). Acceleration measured on the rail while the wheel passes over elements 20÷15 (about 2 m span) instead contains energy from instants before the entering of the wheel on the modeled section and it is therefore neglected;
4. the rail acceleration frequency spectrum obtained by simulation is compared to the measured one. Their difference represents the estimation error. For each 1/3 octave band a gain, to be applied to the input force, is computed to reduce to a minimum the difference between the real and the estimated

responses;

- 30 signals with the corrected frequency content and random phase are applied to the system, iterating the procedure from step 2.

Process is stopped when the error mentioned at step 4 is within a fixed limit. Usually two iterations proved to be sufficient.

### 3.2 Simulation of the track response

The system considered here consists of 20 inputs and 4 outputs. When a wheel passes over the modelled track span, each element in fig. 1 is subjected to distinct segments of the contact force, with an amplitude that depends on the speed. Under the hypothesis of linearity the principle of superposition holds, and the response of the system can be easily computed as the sum of the outputs due to the application of different forces applied to the different elements.

The excitation of the system through the application of distinct signal segments to rail elements is anyway too rough. To obtain a smoother and more realistic result a triangular window has been superimposed to each segment, such that adjacent segments are weighted in a linear way. The underlying hypothesis is that the FRFs change linearly with space between two adjacent points. Exclusively for software implementation reasons a different but almost equivalent approach was used: FRFs were retained as constant for each element, while the force was distributed such that it entirely acts over an element when it is just over its centreline and decreases to zero moving to the centreline of the adjacent element (fig. 2). These weighted force segments were fed to the corresponding elements of the ARMAX model of the track.

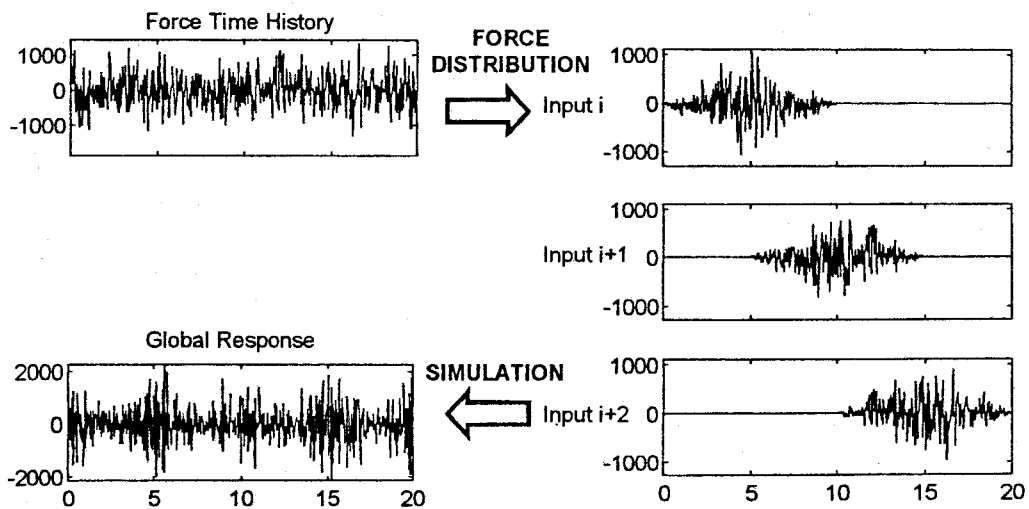


Fig. 2. Application of the estimated contact force to the system. The signal is linearly distributed on adjacent elements as explained in the text body.

## 4. SOME EXAMPLES OF FORCE RECONSTRUCTION

The developed algorithm is particularly fast and stable. Some results shown here are obviously relative to Italian rolling stock but, as almost all kinds of passenger rolling stock passed on the instrumented section, only some selected results are presented. Trains have been selected by using the following:

- only trains with “blocked composition” are analyzed here, ensuring an equal mileage run by all of the wheels;
- only trains with rigorously constant speed have been chosen (a critical condition as almost all trains accelerate in the measuring site) such that energy analysis is formally correct;
- whenever possible results are compared for similar rolling stock running at different speeds;

- the ETR450 (8 EMU, known as “Pendolino” of the first generation) and ETR460 (9 EMU, the new “Pendolino” generation) were chosen as examples of homogeneous light rolling stock for fast services (up to 250 km/h);
- the ALe601 (up to 8 elements, but typically 4÷5 elements EMU) have some motorized elements tread braked and some coaches disc braked; it was developed in the ‘60s as luxury trains with very good ride characteristics.

First of all the reconstruction of the rail acceleration in the V1 position is compared to the measured acceleration (fig. 3 left). The reconstruction process should converge to measured results in a few iterations: it is possible to see how just after two iterations the results can be considered extremely good (the initial resulting acceleration is meaningless as it comes from a flat spectrum force and it is not shown). The force necessary to obtain such an acceleration spectrum is shown in fig. 3 right where, starting from the constant average level spectrum explained in par. 3.1, the final force has a notably different shape.

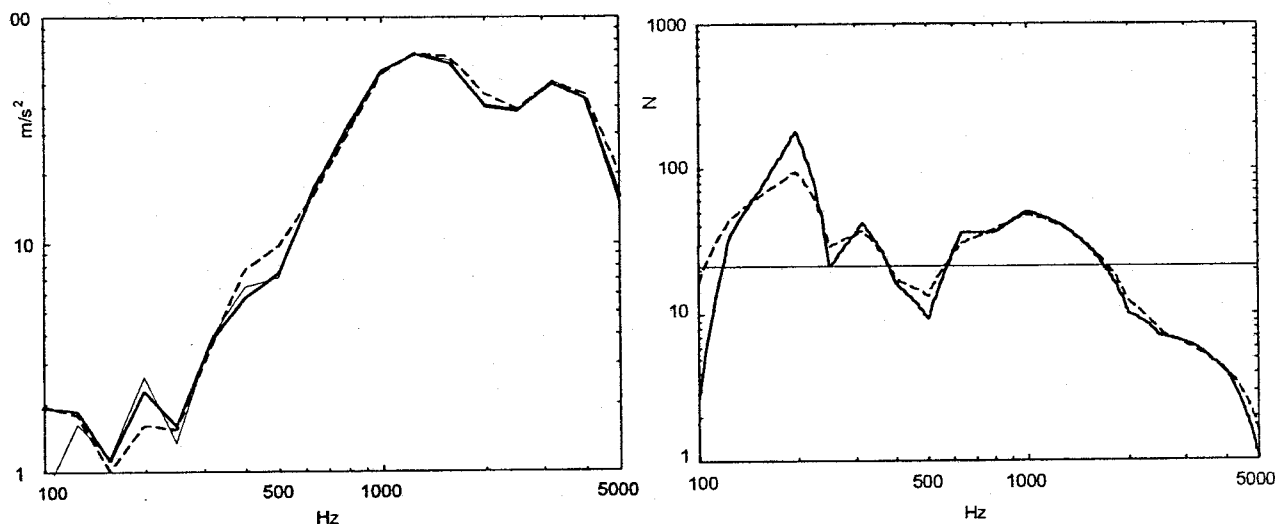


Fig. 3. Acceleration in position V1 and reconstructed forces for a wheel of an ETR450 train running at 80 km/h. Measured (thin line) and estimated (1<sup>st</sup> iteration dashed line, 2<sup>nd</sup> iteration thick line) 1/3 octave acceleration (left). Rolling contact forces narrow band spectra (log-log plot): initial pseudo-random force (thin line), 1<sup>st</sup> iteration (thick dashed line), 2<sup>nd</sup> iteration (thick continuous line) (right).

As previously stated, the presence of four outputs allows the estimation of the statistical reliability of the reconstructed forces.

Forces reconstructed by using different accelerometers for a given wheel are very similar, confirming the validity of the proposed procedure. Fig. 4 left shows as an example the force reconstructed from V1÷V4 for an ETR450 wheel; it must be noted that, generally speaking, signals reconstructed by using V4 signals are sometimes quite different from V1÷V3 signals, but this can be due to the substantially different dynamic behaviour of the rail at point V4 and to the transverse sensitivity of the transducer that certainly influences this measurement. For subsequent analyses the average of the four reconstructed forces was assigned to each wheel. Forces reconstructed for nominally equal wheels are quite similar (fig. 4 right), as only at some frequencies the confidence interval increases.

Reconstructed contact forces for different rolling stock are compared in fig. 5. While the trend remains basically the same, showing some lobes with an overall decrease at the higher frequencies, important differences are evident between tread- and disc-braked rolling stock. Rolling forces increase with speed as expected.

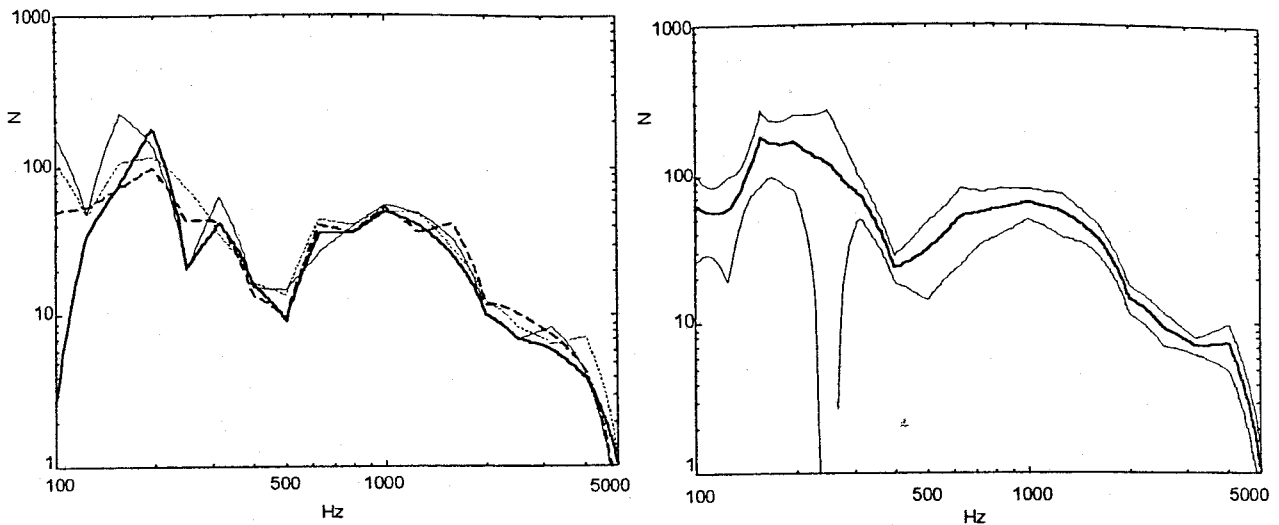


Fig. 4. Contact force narrow-band spectrum (log-log plot) for a wheel of an ETR450 train running at 80 km/h estimated from V1 (thick continuous line), V2 (thick dashed line), V3 (thin continuous line) and V4 (thin dashed line) accelerations (left). Mean value and confidence interval  $\pm\sigma$  for all the wheels of the ETR450 train. The force of each wheel is the average of the forces reconstructed by V1-V4 accelerometers (right).

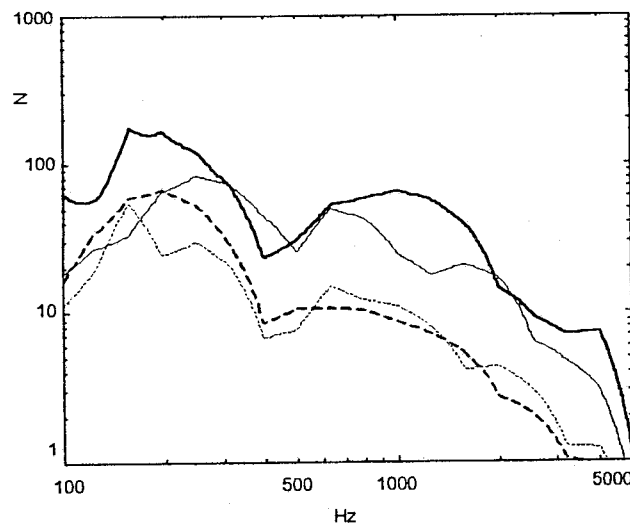


Fig. 5. Comparison of average rolling contact forces for an ETR450 train running at 80 km/h (thick continuous line), an ETR460 train running at 40 km/h (thick dashed line), an ALe 601 train running at 55 km/h (thin continuous line: tread braked, thin dashed line: disc braked)

## 5. CONCLUSIONS

The results obtained with a new procedure for rolling contact force reconstruction based on time histories analysis and on autoregressive modeling of the track are particularly promising. Results are both stable and plausible, and important differences for different rolling stock are depicted.

The use of the new procedure can be useful as an indirect method for wheel tread diagnostics, for the optimization of wheel tread and web shapes and for the generation of feasible controlled forces in laboratory bench noise emission tests.

An extensive analysis will be performed on numerous acquired data; resulting forces, particularly low, mainly depend on the limited line speed, and a new measurement campaign to consider train running at higher speeds will be necessary. It is expected that contact force energy content at high frequencies will increase with speed, coherently to the increase in noise emission. It is important to highlight that an increase in the speed, and a corresponding reduction in time history length, does not represent a problem for the procedure for the particular time-based signal analysis.

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